

# Quantum correlations in twin beams of light

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# “twin beam” correlations

Quantum correlations play an important role in the field of quantum optics (and in many other areas, like cold atom studies).

A very particular part of this field concerns “twin beams” of light – two beams that have correlations that are stronger than “classical” physics allows. Such beams require nonlinear processes to generate these correlations and there are a number of ways in which we can try to measure the correlations.

This is not a universally useful set of things to know... On the other hand, understanding correlations in this context is useful in many other situations as well!

# 5 lectures on correlations in twin beams

**We will discuss what twin beams are, how to generate twin beams, and how to characterize them.**

**Because I have worked with nonlinear optics, and in particular, 4-wave mixing in Rb vapors, that is what you will mostly get for examples.**

**As we go you will see more complex (interesting?) examples of how twin beams are used, in interferometry, for example, and we can discuss how “twin beams” of atoms can be produced via 4-wave mixing in atoms.**

**You are invited to dream up even more interesting ideas!**

# Lecture 1 outline

nonlinear optics

generating twin beams

4WM as well as PDC

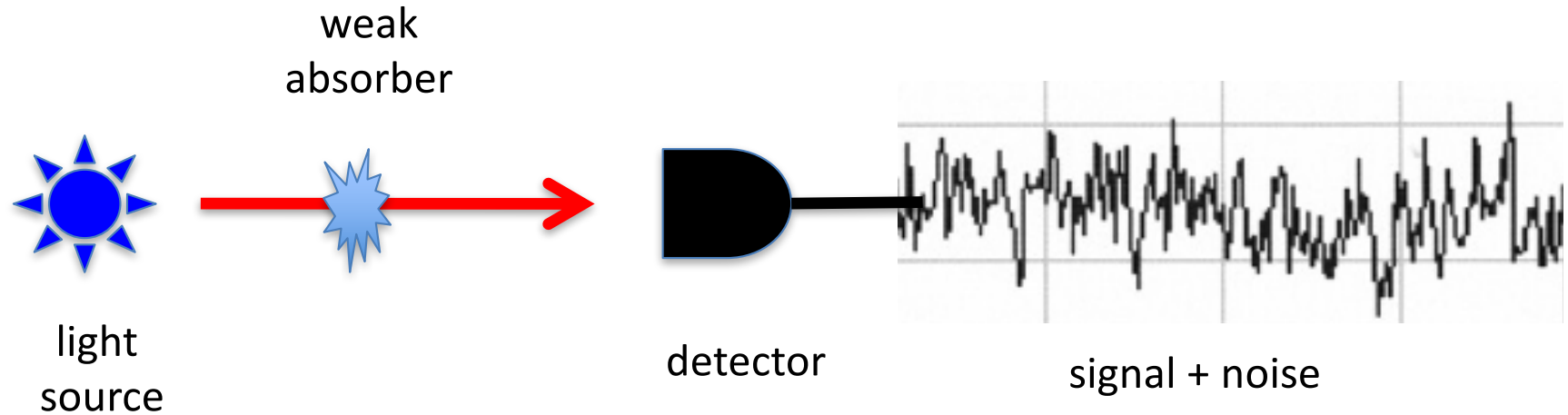
squeezed light (single mode)

2-mode squeezing versus multi-spatial-mode 2-mode squeezing  
noise

measuring noise/squeezing

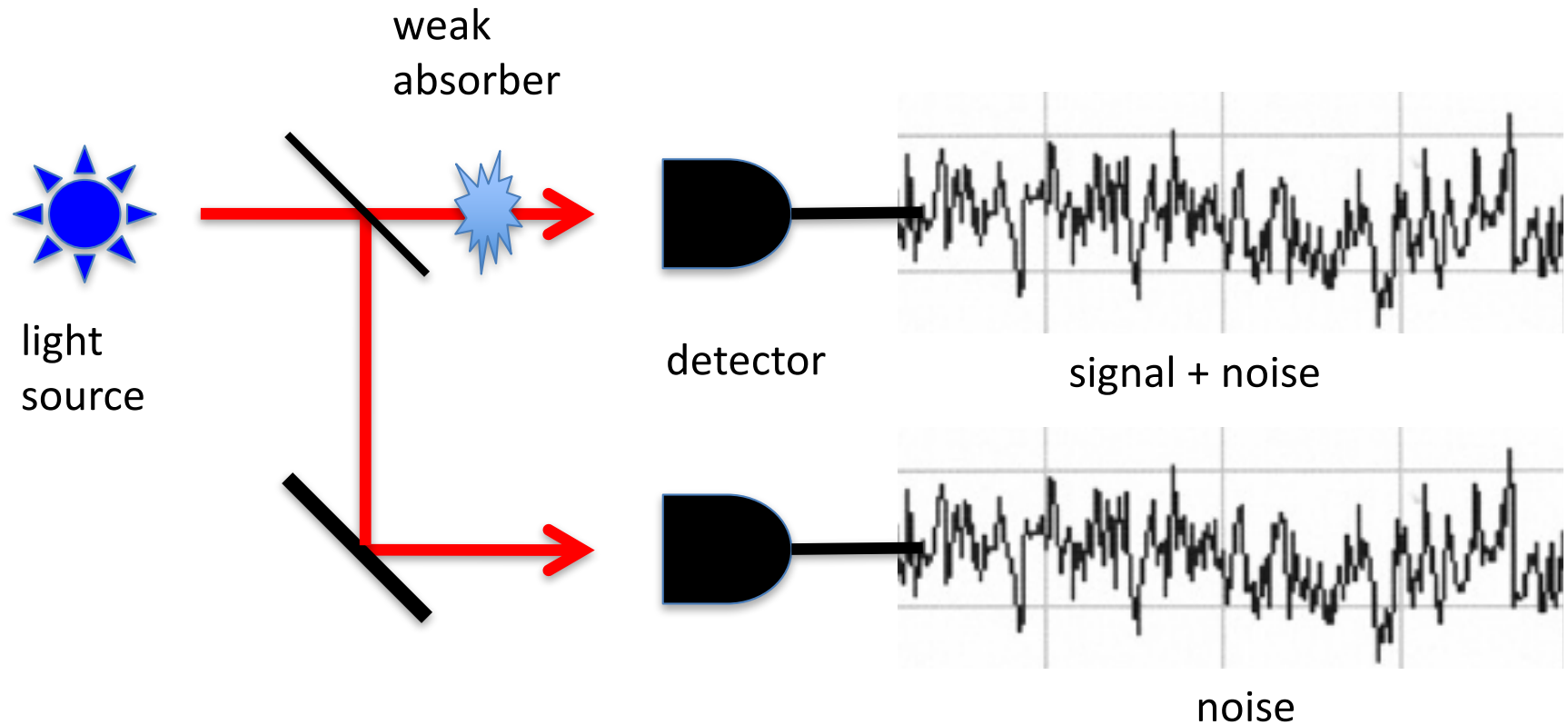
homodyne detection

# making use of correlations



A weak signal, buried in noise, is hard to detect.

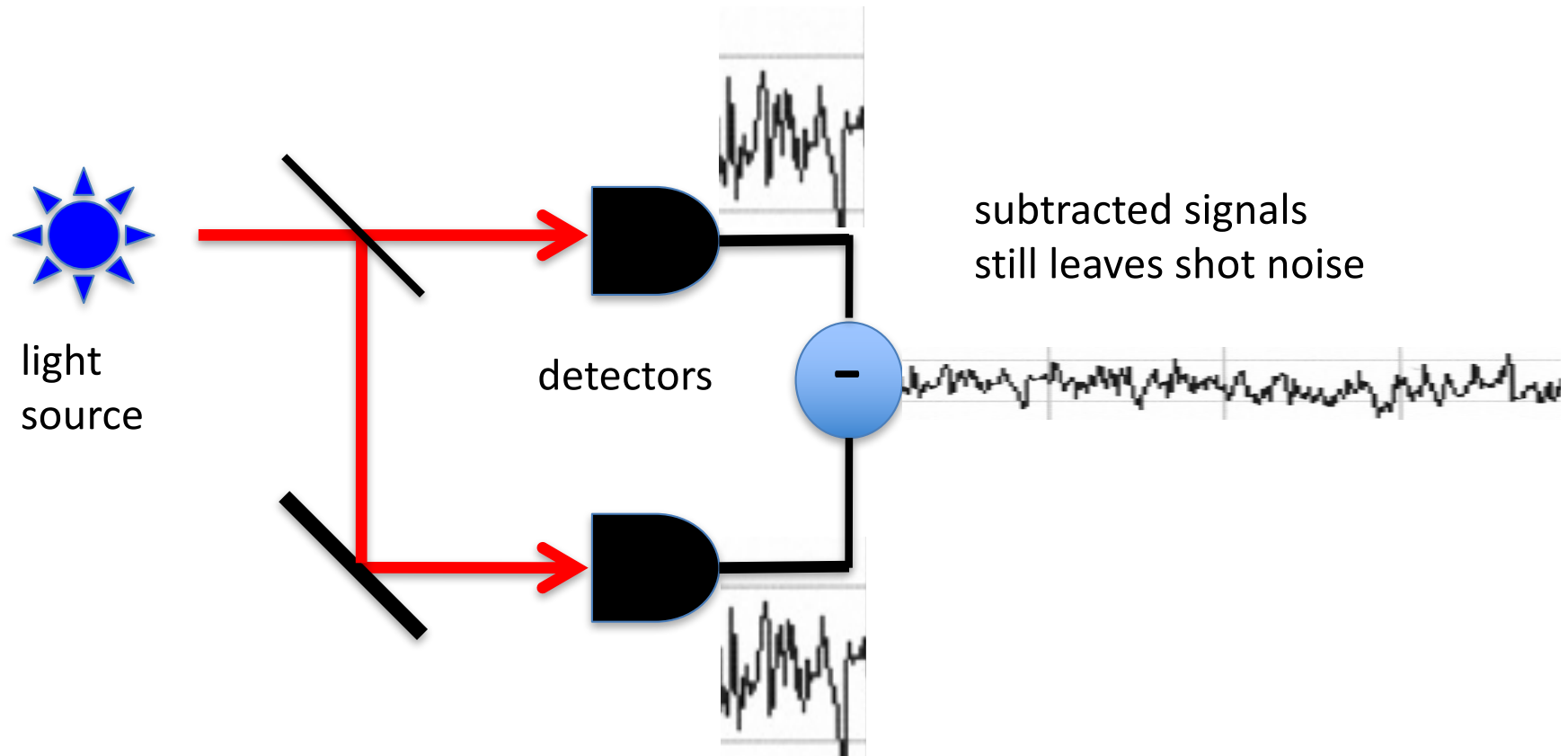
# the advantage of classical correlations



If we *exactly duplicate* the fluctuating probe, then subtract the noise, it is easier to detect the signal.

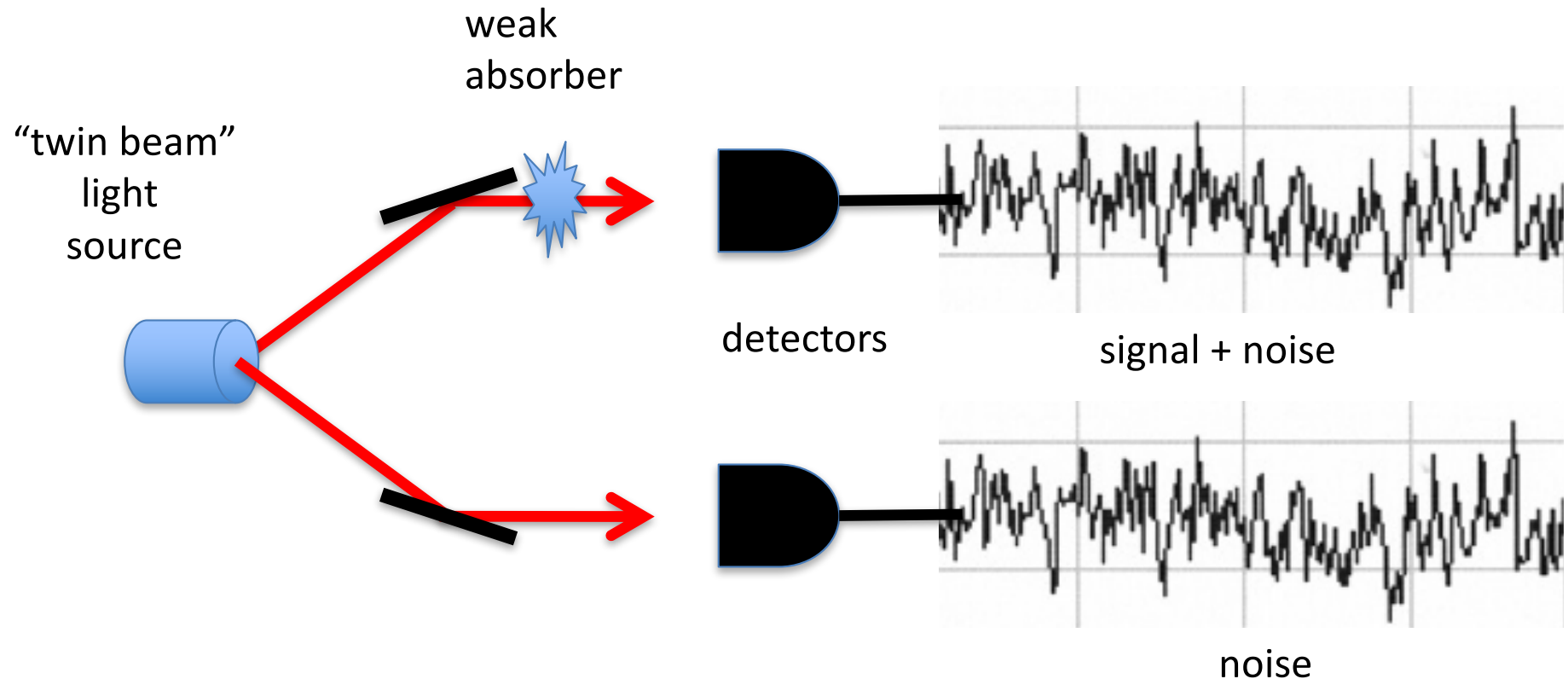


# the PROBLEM of classical correlations



A beamsplitter or half-silvered mirror can “duplicate” only the classical noise on the beam; the “shot noise” due to photons “randomly deciding” which way to go is uncorrelated and adds.

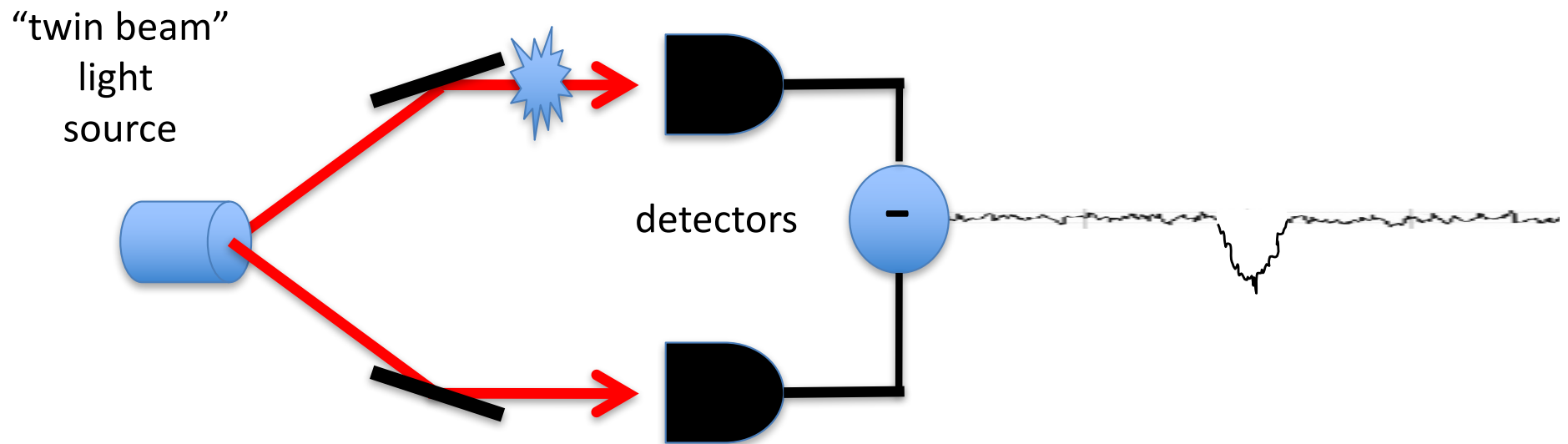
# the advantage of quantum correlations



If we duplicate the fluctuating probe, *including* its quantum noise, then subtract, it is even easier to detect the signal.

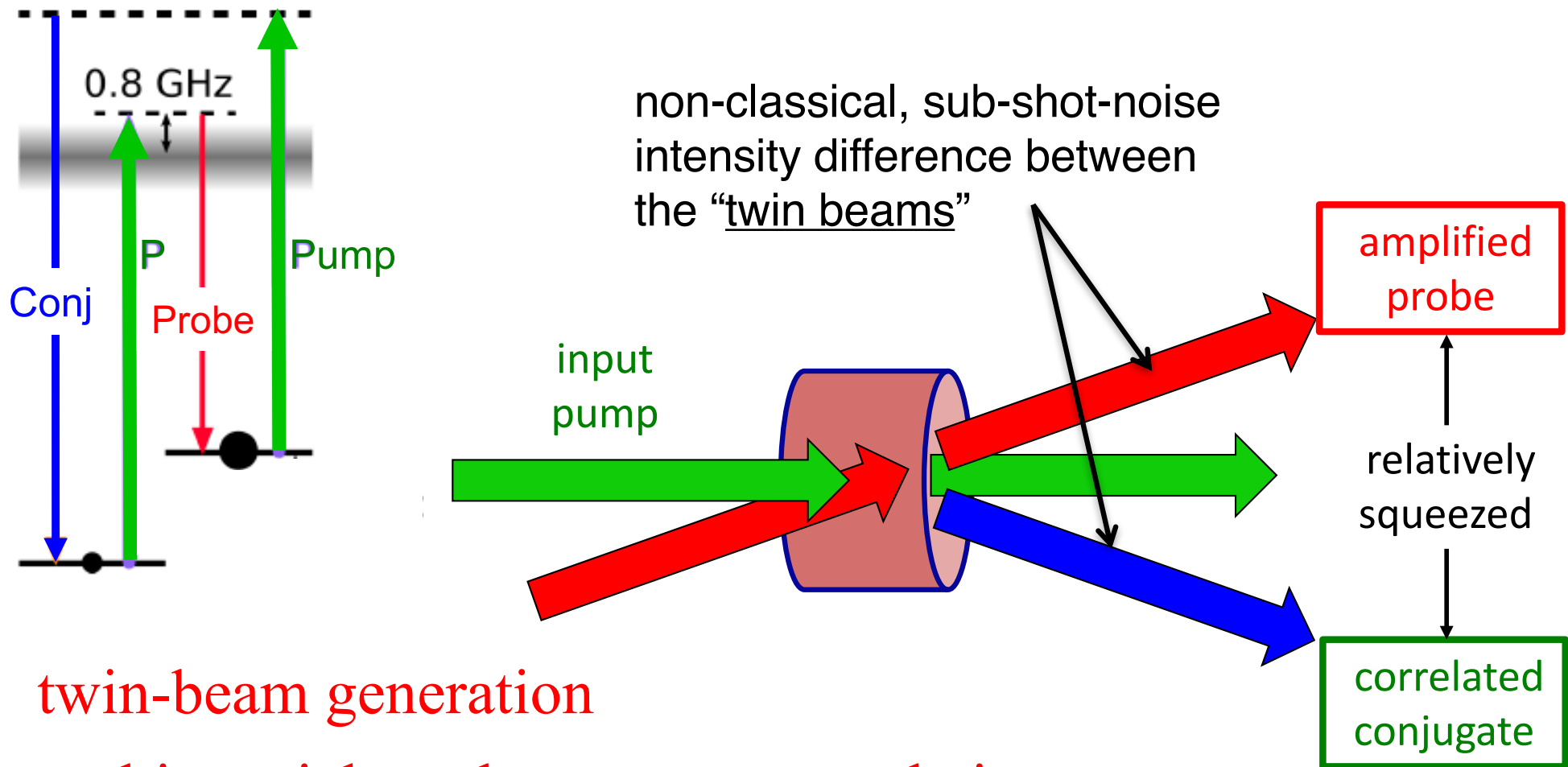


# the advantage of quantum correlations



A “twin beam” squeezed-light source will duplicate the fluctuating probe, including any quantum noise. Thus we can subtract even the quantum noise, and detect previously undetectable signals.

# 4-Wave-Mixing in the quantum limit



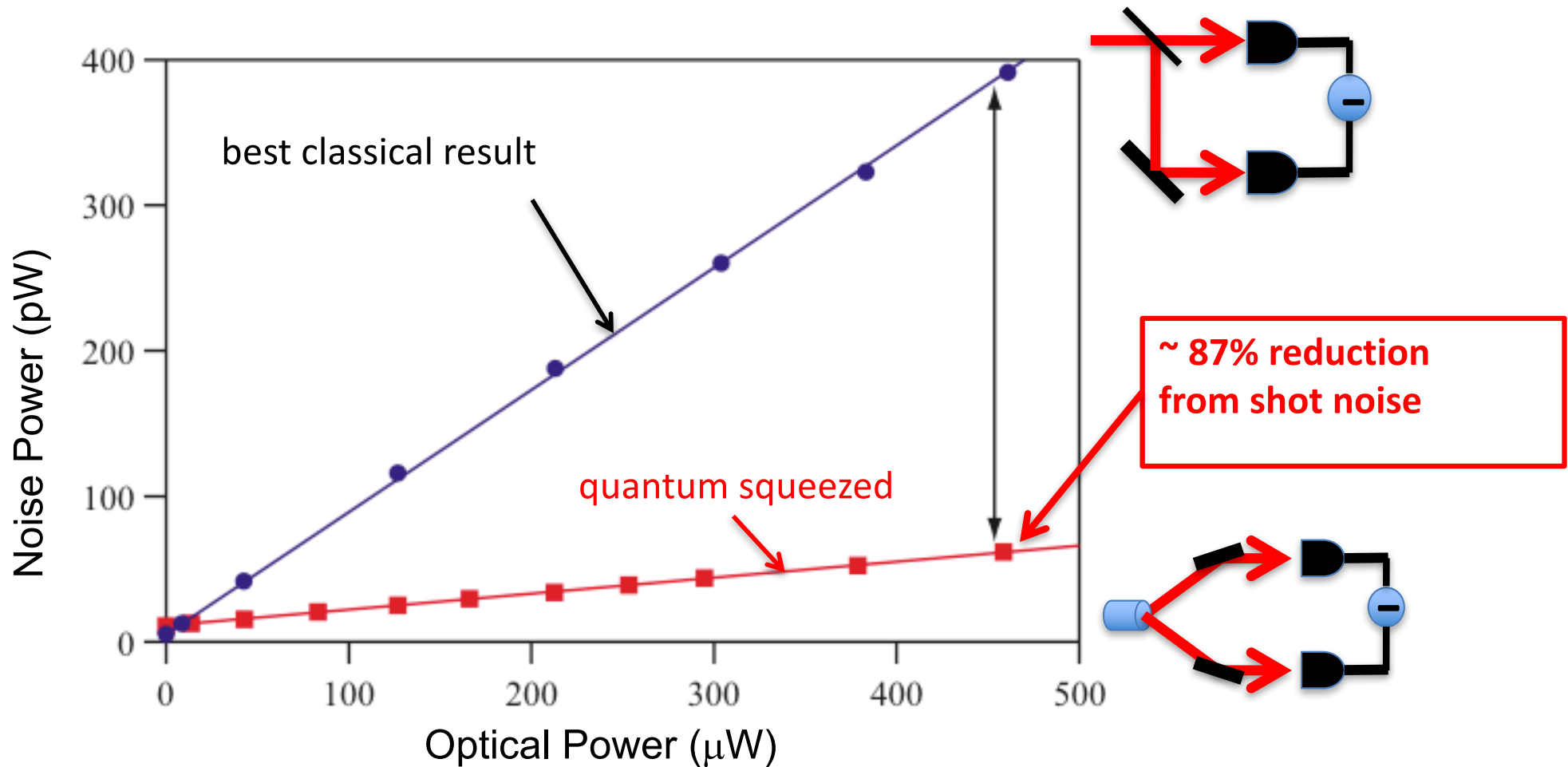
- twin-beam generation
- multi-spatial-mode quantum correlations (quantum imaging)
- acts as a phase insensitive amplifier (PIA)

# The heart of the experiment



This inexpensive, low-tech device helps produce cutting-edge quantum squeezed light!

# twin-beam squeezing



photodiode difference measurement shows  
intensity-difference squeezing

# how do quantum correlations help?

It allows us to avoid some of the limitations of classical light on resolution and noise in optical systems.

It is useful in position sensing, interferometry, and image processing.

**How do we get quantum correlations?**

# nonlinear optics

- frequency converter:

couple 3 optical fields to make a 4th:

$$E_4 = \chi^{(3)} E_1 E_2 E_3$$

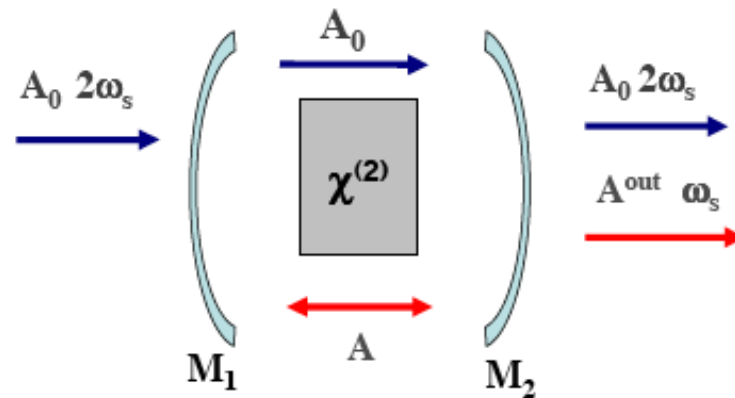
- requires that energy and momentum are conserved between input and output

energy:  $\omega_1 + \omega_2 = \omega_3 + \omega_4$

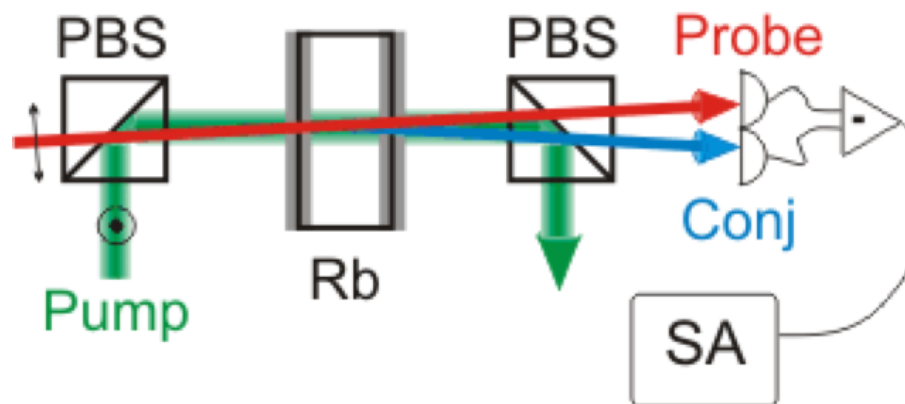
momentum:  $k_1 + k_2 = k_3 + k_4$

If we are careful about how we do it, inserting 3 fields gets us a 4th with some “magic” properties

# 4wm vs. OPOs ...or... $\chi^{(3)}$ vs $\chi^{(2)}$ media



OPO  
(optical  
parametric  
oscillator)



four-wave mixing (4WM)  
(count 2 pump photons in  
the 4 waves!)

# photon correlations vs. squeezing

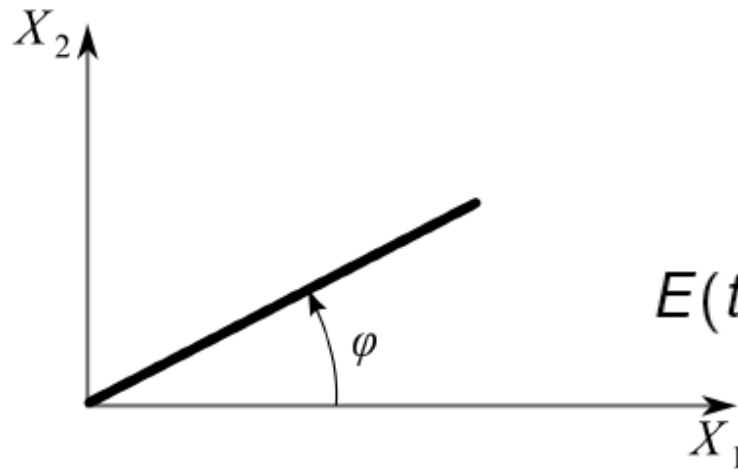
PDC is often generated with low efficiency and so “digital” single-photon correlations are studied. Properties such as the polarization are often the most important.

4WM usually has a much stronger nonlinearity and the light produced is often at a high enough intensity that a continuous variable description (“squeezed light”) is more appropriate. Often continuous parameters such as the phase of the beam are important.



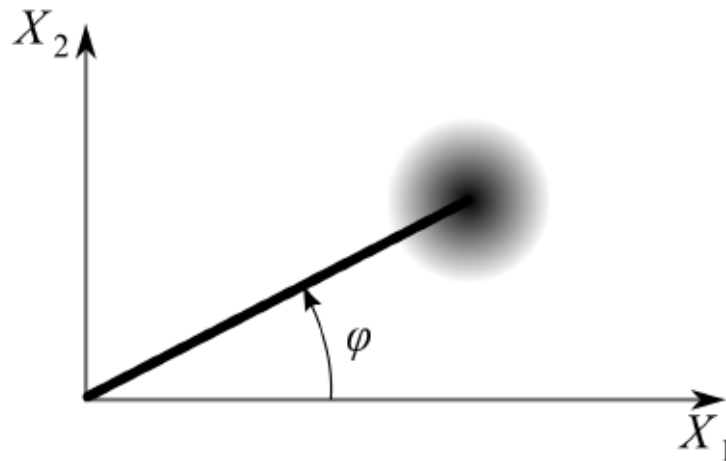
# light

amplitude and phase  
or  
quadratures



$$E(t) \propto X_1 \cos(\omega t) + X_2 \sin(\omega t)$$

quantum version  
with noise

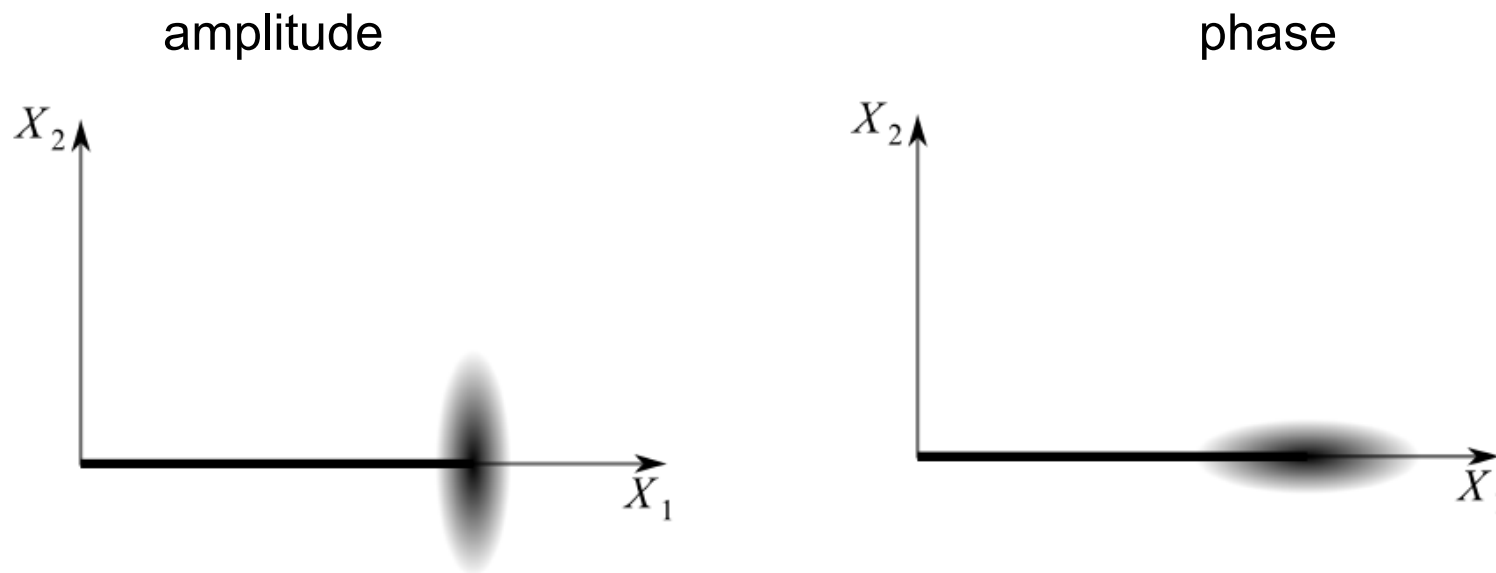


coherent states have  
minimum-sized,  
round fuzz-ball

# squeezed light

what does it mean for light to be “squeezed”?

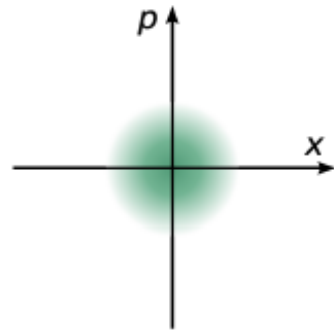
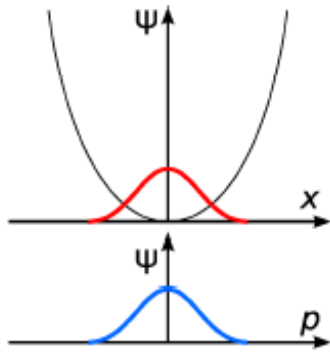
classical fields have symmetric noise



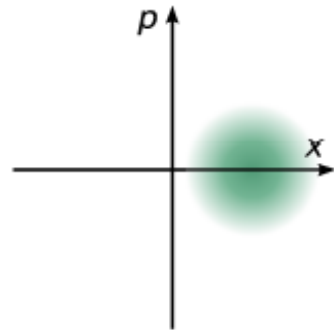
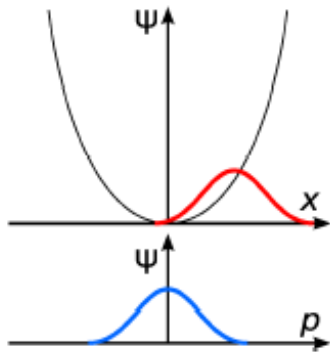
- different kinds of squeezing:
- amplitude/phase or quadrature

$$E(t) \propto X_1 \cos(\omega t) + X_2 \sin(\omega t)$$

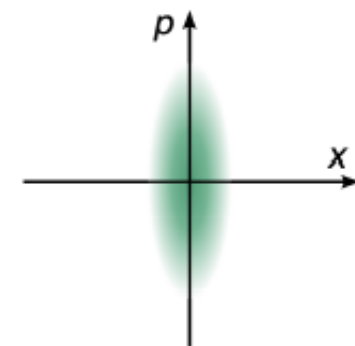
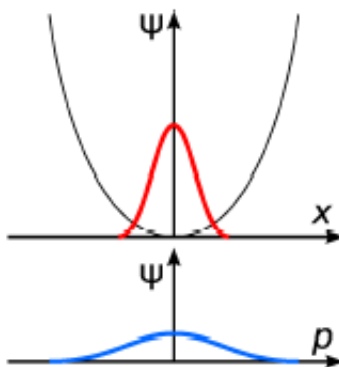
# Single-mode squeezing



- Mode of frequency  $\omega \equiv$  harmonic oscillator of frequency  $\omega$
- $\hat{x} \rightarrow \hat{X}$   
 $\hat{p} \rightarrow \hat{Y}$   
with the rotation taken out



- Coherent state:  
 $\langle \Delta \hat{X}^2 \rangle = 1$   
 $\langle \Delta \hat{Y}^2 \rangle = 1$

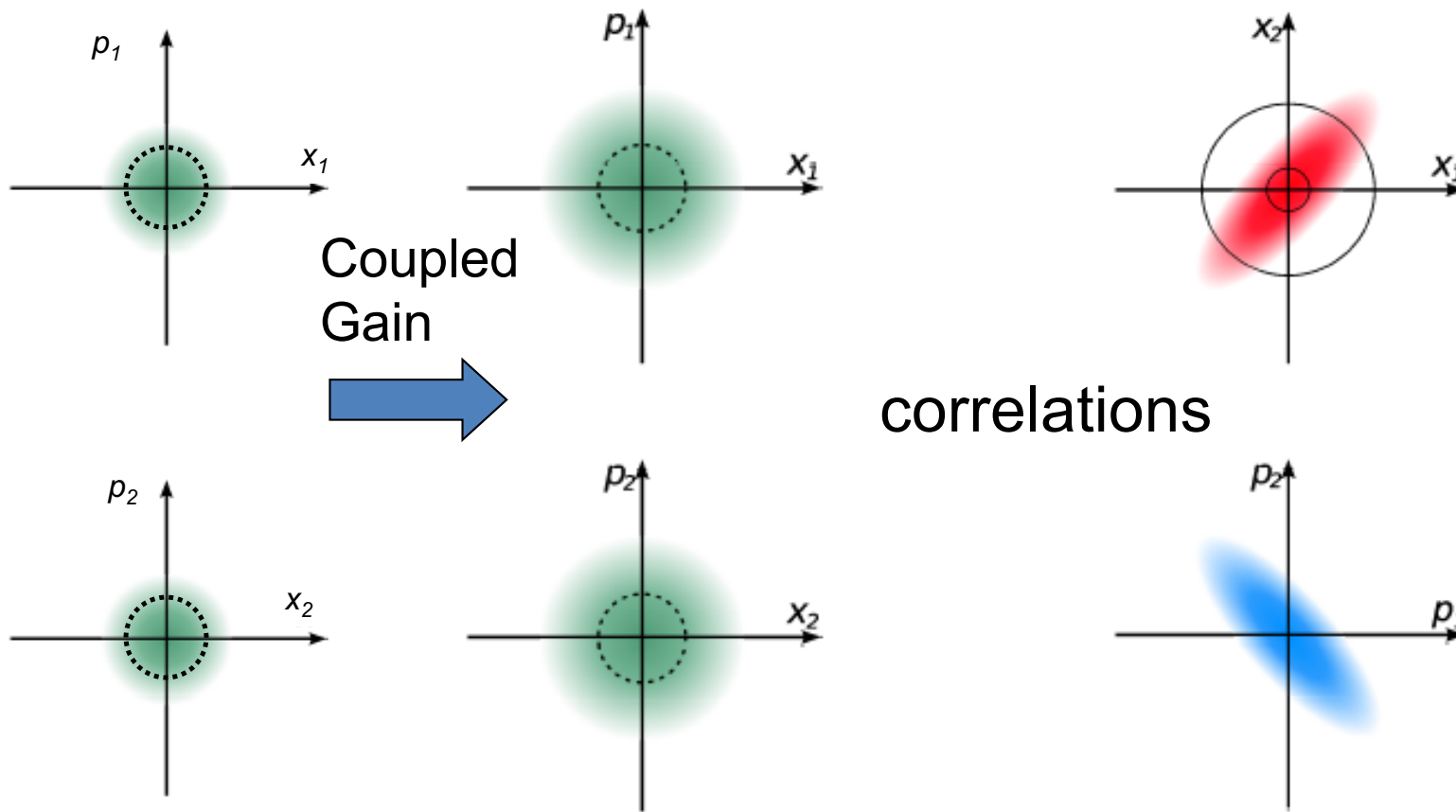


- Squeezing:  
 $\langle \Delta \hat{X}^2 \rangle < 1$   
 $\langle \Delta \hat{Y}^2 \rangle > 1$
- Pairs of photons
- Generalize to bright beam

**we will mostly talk about  
“twin-beam squeezing” and not  
(single mode) “quadrature squeezing”**

Instead of amplitude and phase of a single beam, the interesting variables here are the intensity difference and the phase sum of these “twin” beams.

# Two-mode squeezing: phase-insensitive amplifier

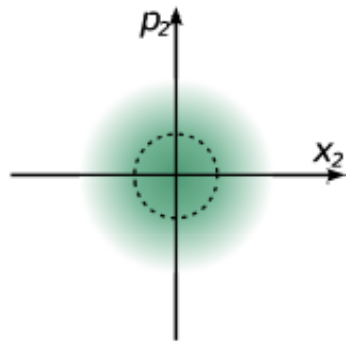
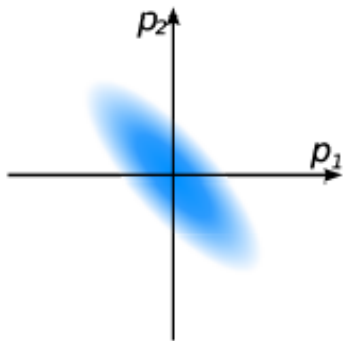
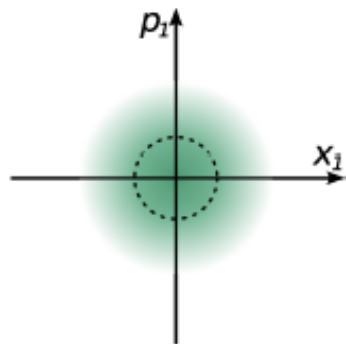
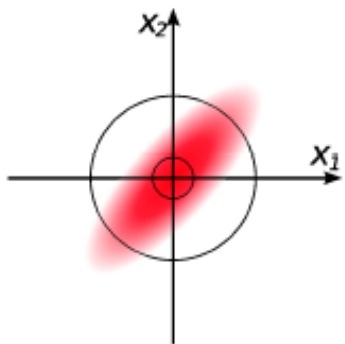


two vacuum  
modes



two noisy, but entangled,  
vacuum modes

# Squeezing quadratures



- Generalized quadratures:

$$\hat{X}_- = \frac{1}{\sqrt{2}}(\hat{X}_1 - \hat{X}_2)$$

$$\hat{Y}_+ = \frac{1}{\sqrt{2}}(\hat{Y}_1 + \hat{Y}_2)$$

- Squeezing:

$$\langle \Delta \hat{X}_-^2 \rangle < 1$$

$$\langle \Delta \hat{Y}_+^2 \rangle < 1$$

- Individual modes are noisy:

$$\langle \Delta \hat{X}_1^2 \rangle > 1$$

$$\langle \Delta \hat{Y}_1^2 \rangle > 1$$

# squeezing light

compare to “shot noise”

if we detect  $N$  particles, on average, during a time  $t$ ,  
and the statistics are “random” (Poissonian)  
then the standard deviation of the detected number will be  $\sqrt{N}$

$$\text{Var} = \langle (n - \langle n \rangle)^2 \rangle = \langle (n - N)^2 \rangle = N$$

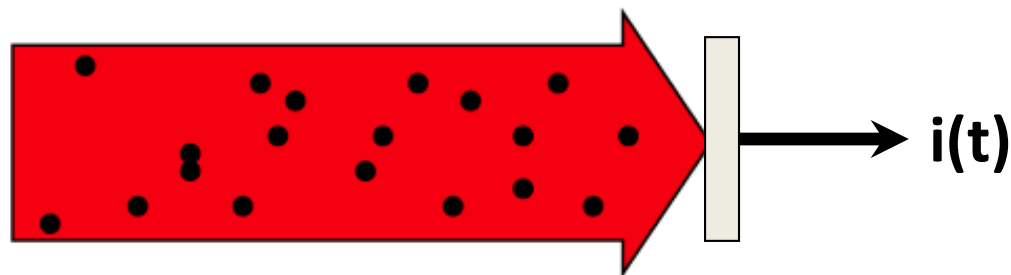
If the variance is more than that it is “super-Poissonian”  
(this is not unusual – “technical noise”)

If the variance is less than that it is “sub-Poissonian”  
(this is unusual - and is a sign of non-classical effects in light)

# “Normal” squeezing (in time)

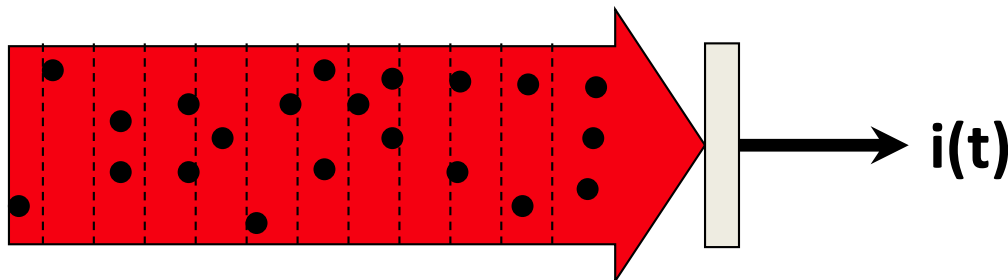
classical or coherent light

random arrivals in time and space



“shot noise”  
random in time;  
no correlations

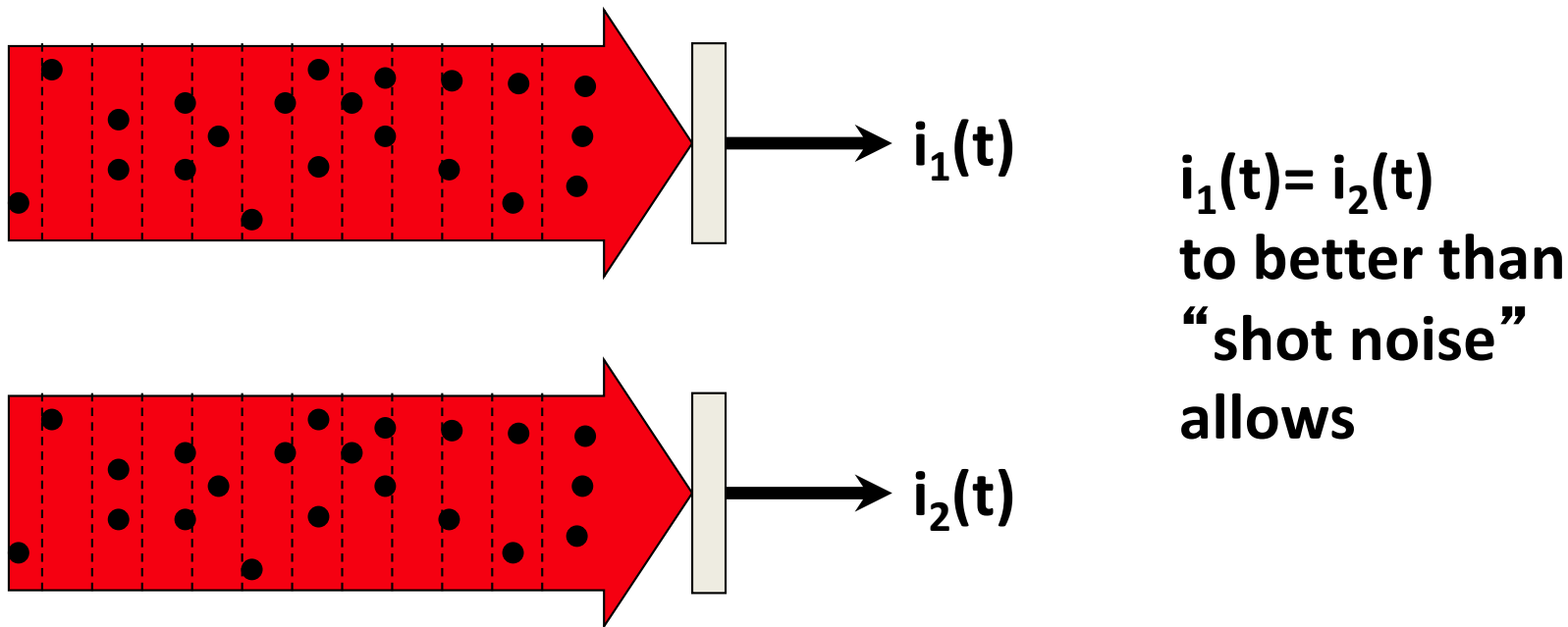
squeezing; ordered in time; reduced fluctuations



can be used to reduce noise in measurements



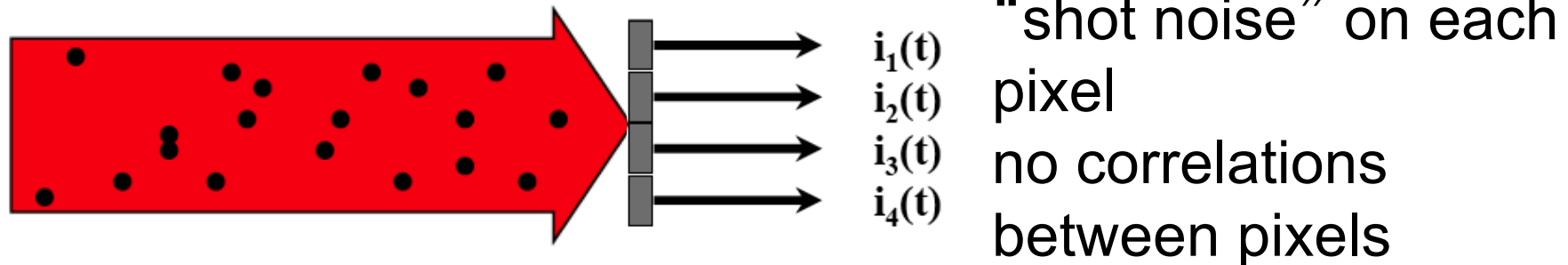
# “Twin beam” or relative-intensity squeezing



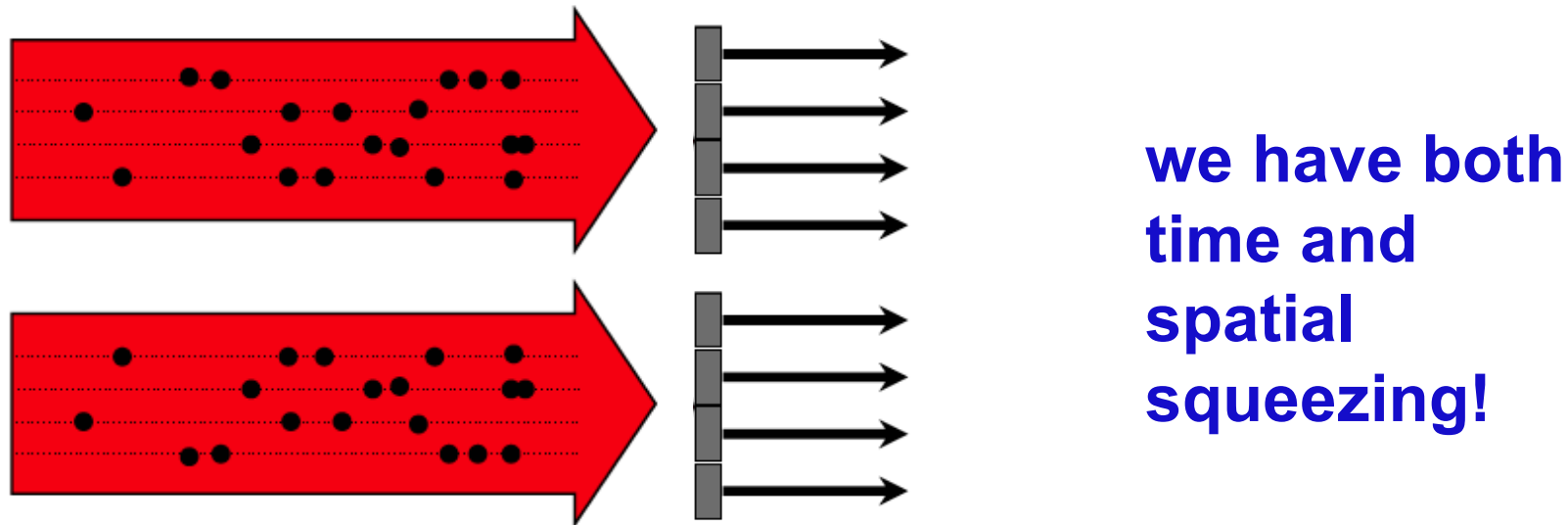
I can't clone a photon, but I can create 2 (nearly) identical photons or beams of them

# spatial squeezing: “quantum imaging”

classical or coherent light  
random arrivals in time and space



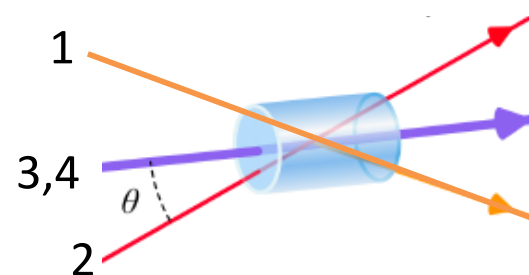
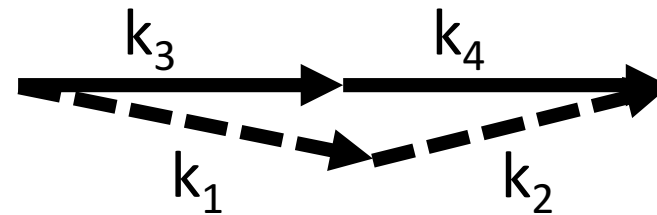
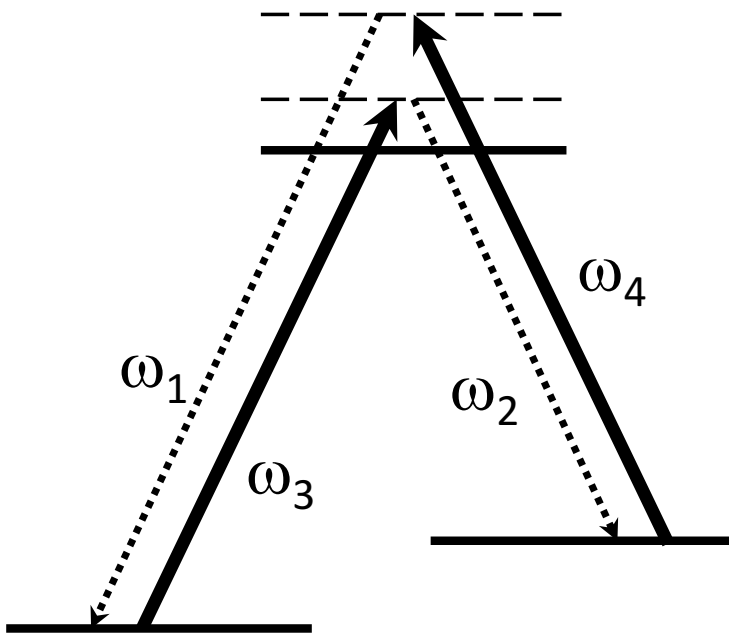
spatial squeezing; ordered in space, correlations between pixels



can be used to reduce noise in spatial information

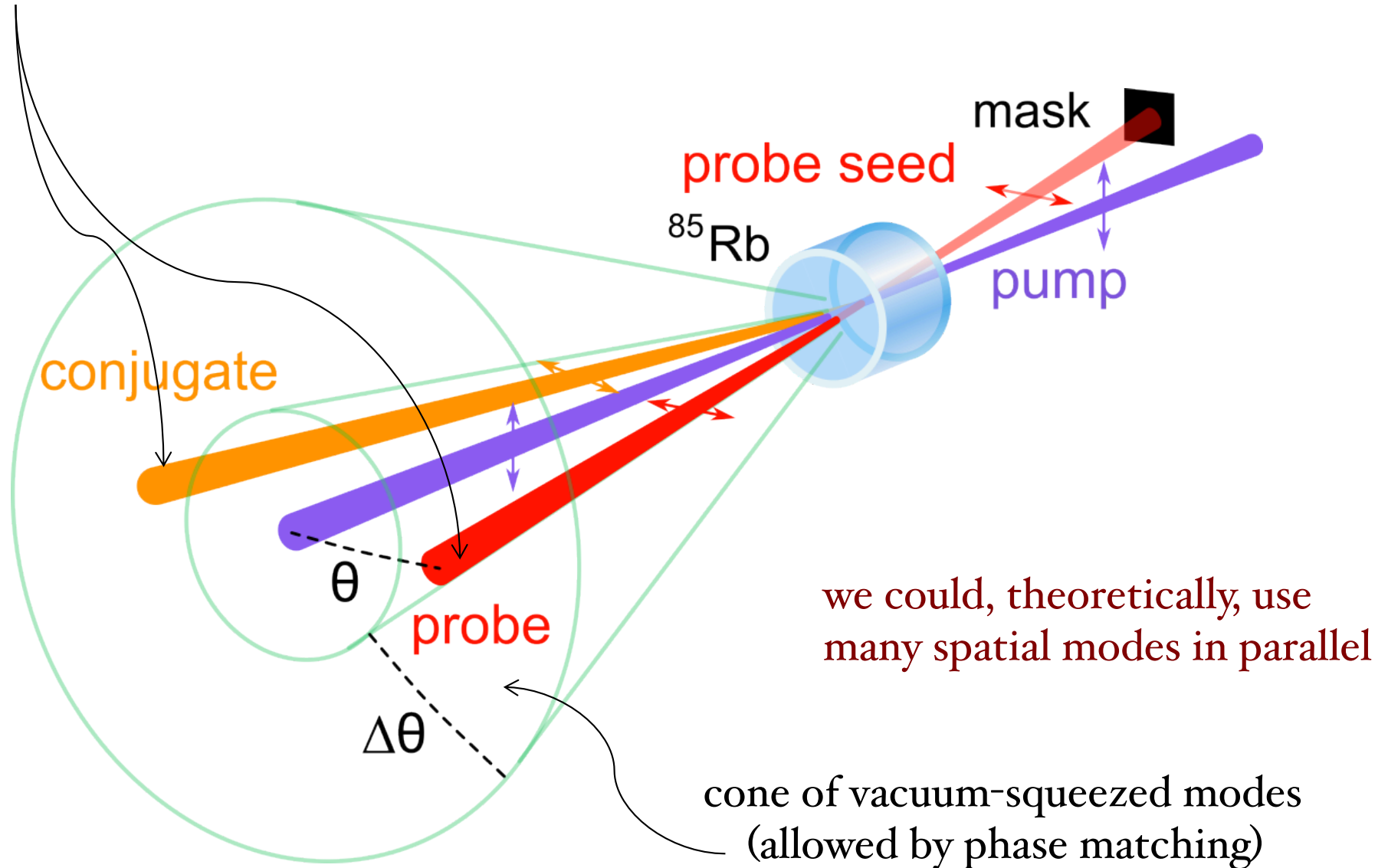
# Phase matching

conserve energy:  $\omega_1 + \omega_2 = \omega_3 + \omega_4$   
conserve momentum:  $k_1 + k_2 = k_3 + k_4$



# multi-spatial mode two-mode squeezing

seeded, bright modes



# squeezed and entangled cats



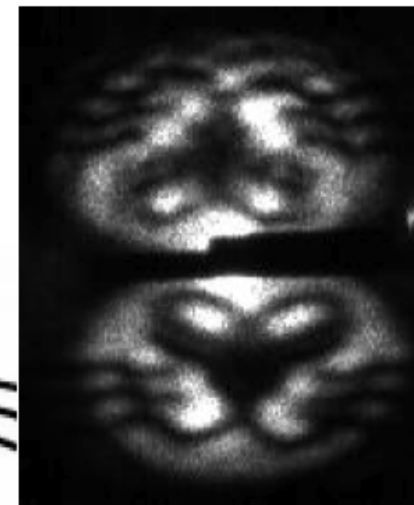
Probe LO



Conjugate LO

local oscillators for  
measurements of 1 dB  
quadrature squeezed  
vacuum

squeezed cats



probe

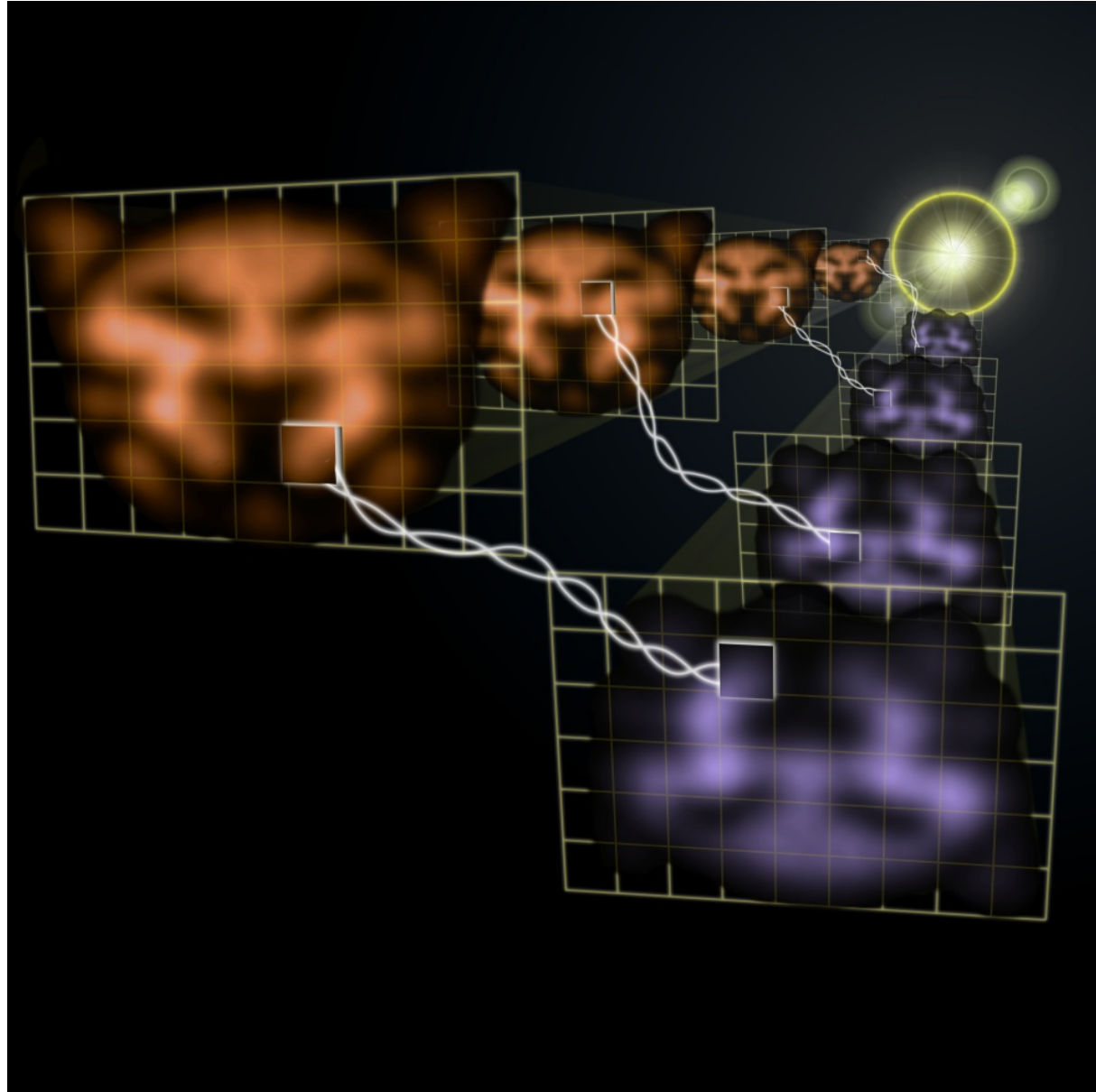
conjugate

bright beams showing  
intensity-difference  
squeezing

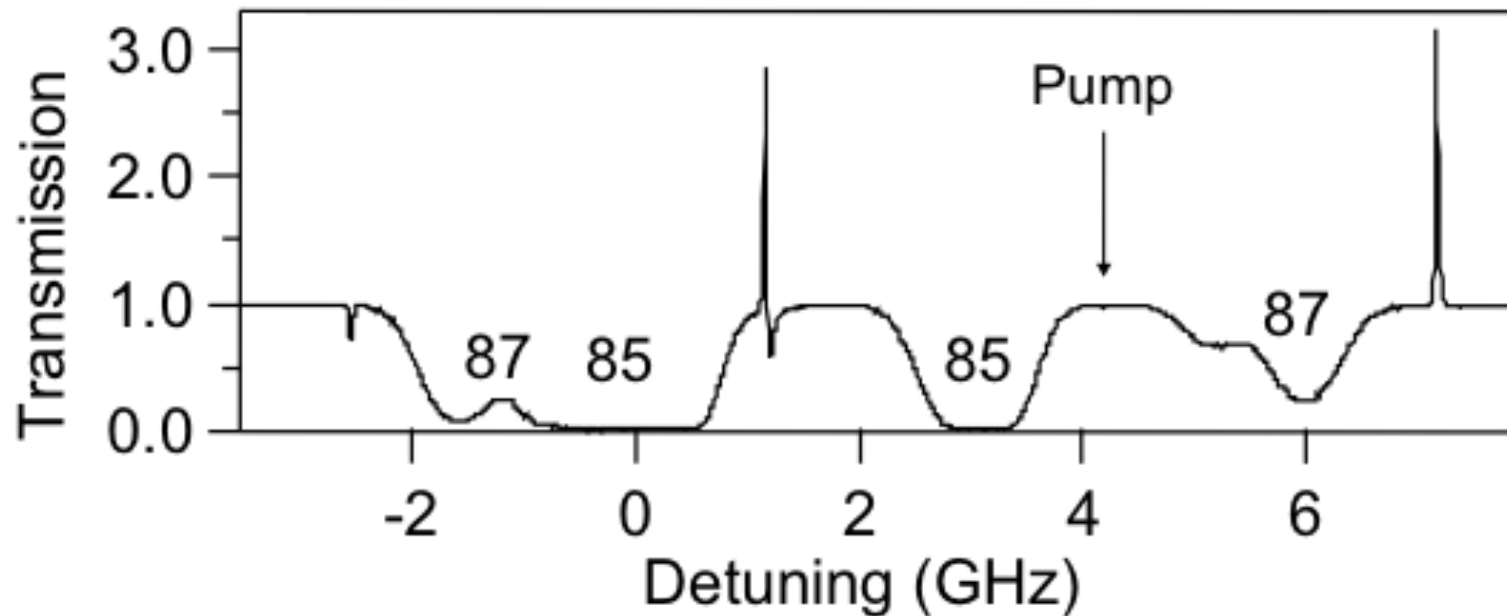
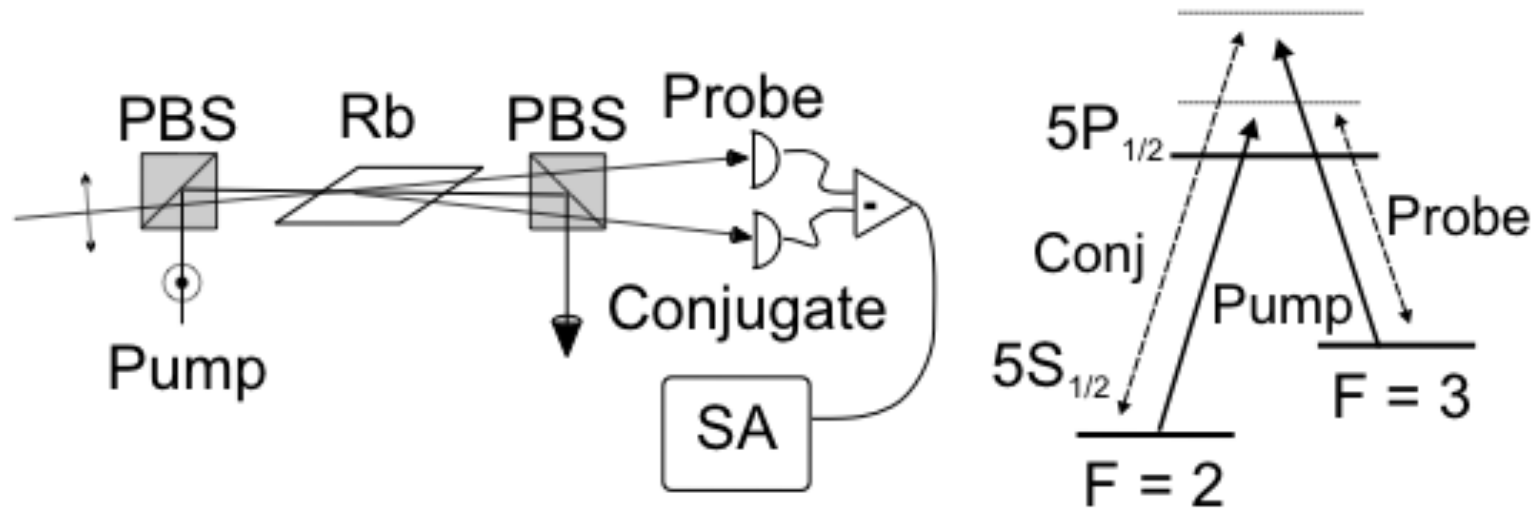


~1 dB “whole image”  
intensity-difference squeezing

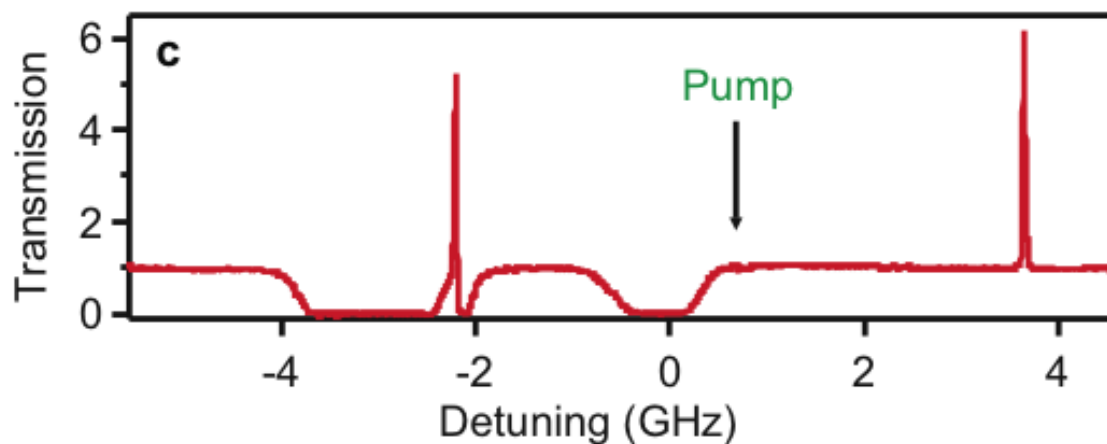
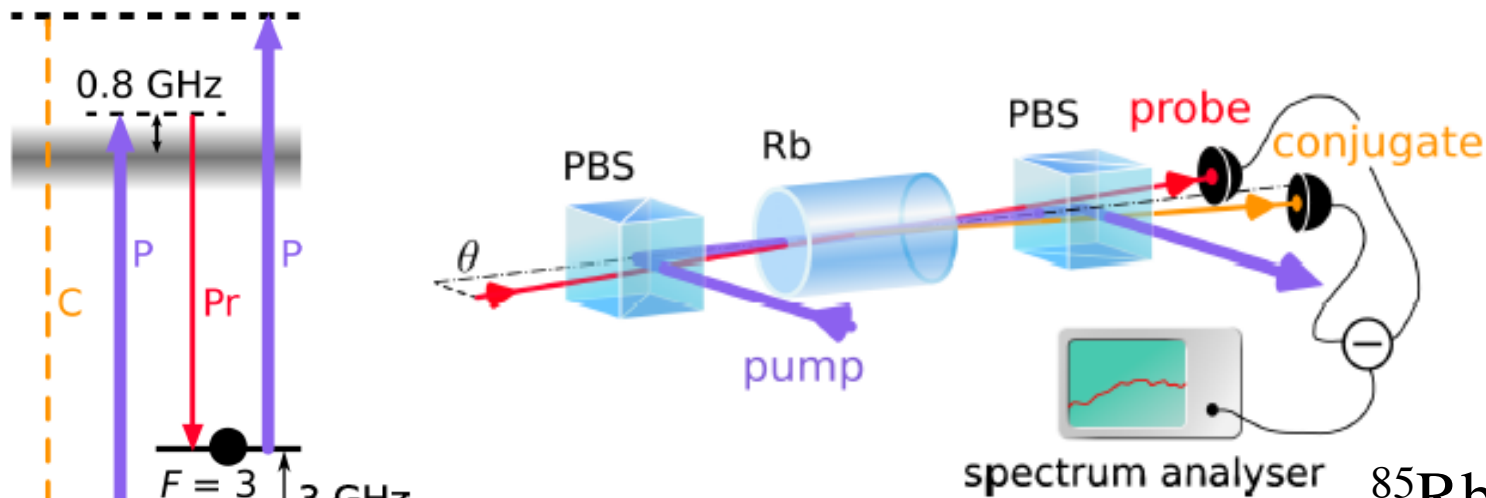
# enhanced graphics!



# Experimental scheme



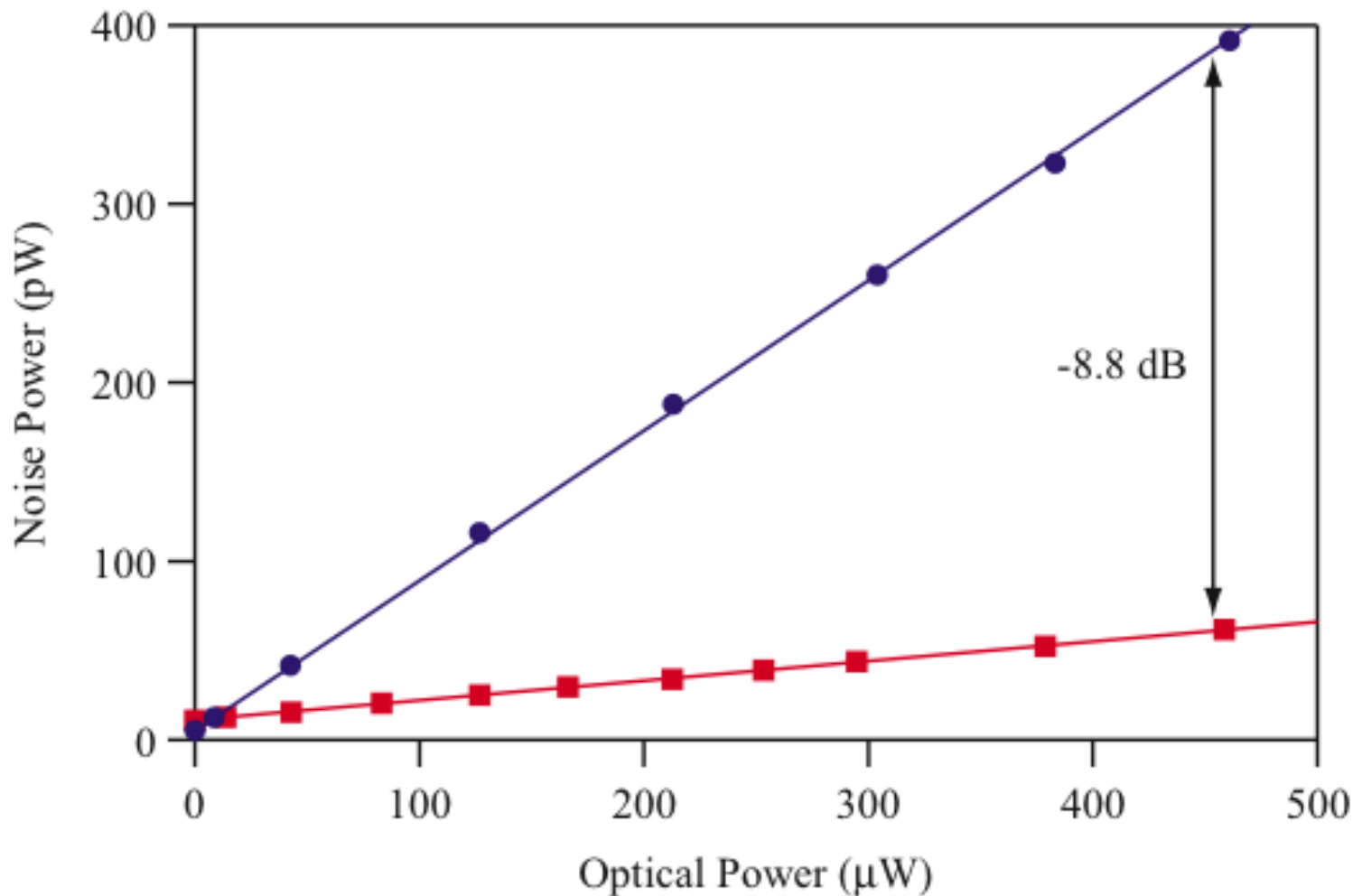
# squeezing from 4WM in hot Rb vapor



$^{85}\text{Rb}$  in a double- $\Lambda$  scheme  
~1 GHz detuned  
~400 mW pump  
~100  $\mu\text{W}$  probe  
narrowband  
no cavity



# strong intensity-difference squeezing measured



1 MHz detection frequency  
RBW 30 kHz  
VBW 300 Hz  
pump detuning 800 MHz  
Raman detuning 4 MHz  
~95% detector efficiency

# noise

- “squeezed light” implies, in some form, reduced fluctuations
- this is usually compared to “shot noise”
- $N$  particles/second  $\Rightarrow$  noise  $\sim N^{1/2}$
- **state of the art;** (linear and log)
- 3 dB = factor of 2; 10% noise = -10 dB
- We have achieved -9.7 dB (~11% of “shot noise”)
- world record (using an OPO): -15 dB (~3%)
- previous best with 4WM in atoms: -2.2 dB
- LIGO will use -6 dB of squeezing in phase II

# measuring squeezing

Squeezing is given as a noise power;  $10 \log_{10}(R)$  where  $R$  is the (linear) noise reduction from shot noise

This comes from an electrical engineering approach to measuring noise power

$R = 0.1 \Rightarrow -10 \text{ dB}$  of squeezing

# Inferred squeezing (in the lab)

If you correct for losses in transmission and detection of photons, and assume the beams are balanced, you can project what might have been measured in a more perfect world...

We infer approximately 12 dB of squeezing at the source in our work.

(from uncertainties and limited by seed light noise)

# inferring squeezing (cont.)

$$R_{\text{meas}} = 1 - \eta + \eta R_0 \quad R \sim \text{linear fractional noise}$$

and  $\eta$  is the detection/collection efficiency

**from -7.1 dB measured**

we infer -10.2 dB at the source

from  $\eta = 0.95(\text{detectors}) \times 0.93(\text{optics}) = 0.89$

from **-9.7 dB and**  $\eta = 0.90$  we infer -12.8 dB

our intensity-difference squeezing is “degraded”  
by the imbalance from an injected signal

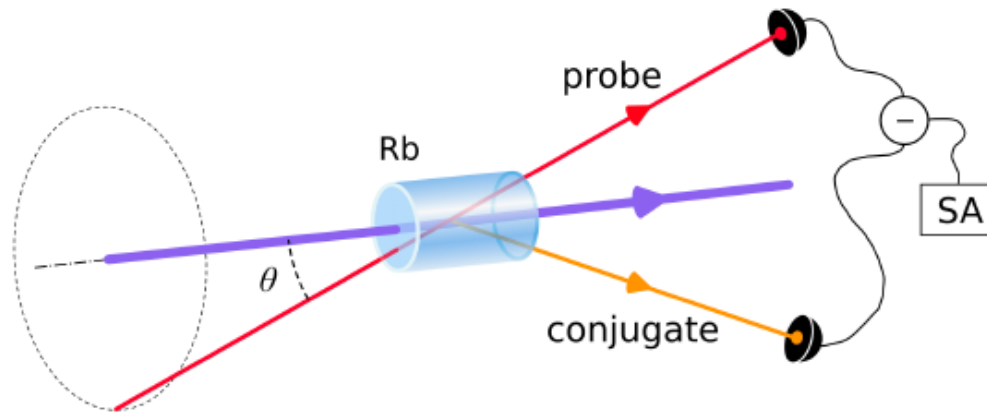
# Correlations vs. Entanglement

Although entanglement is predicted in this system, we have only explored the relative-intensity correlations thus far. Correlations can be on a single variable; entanglement requires a non-commuting pair of variables.

We expect to see phase-sum, as well as intensity-difference squeezing in this system, and a continuous-variable EPR relationship can be established for this system. To show this we need two local oscillators.

Then we can do quantum information processing....

# generating twin beams



correlated: If a particle in beam 1 has spin up, then the corresponding particle in beam 2 has spin down

entangled: I get to choose the spin axis randomly

# homodyne measurements

If we want to make phase-sensitive measurements of a field we need to have a phase reference in our measurement device.

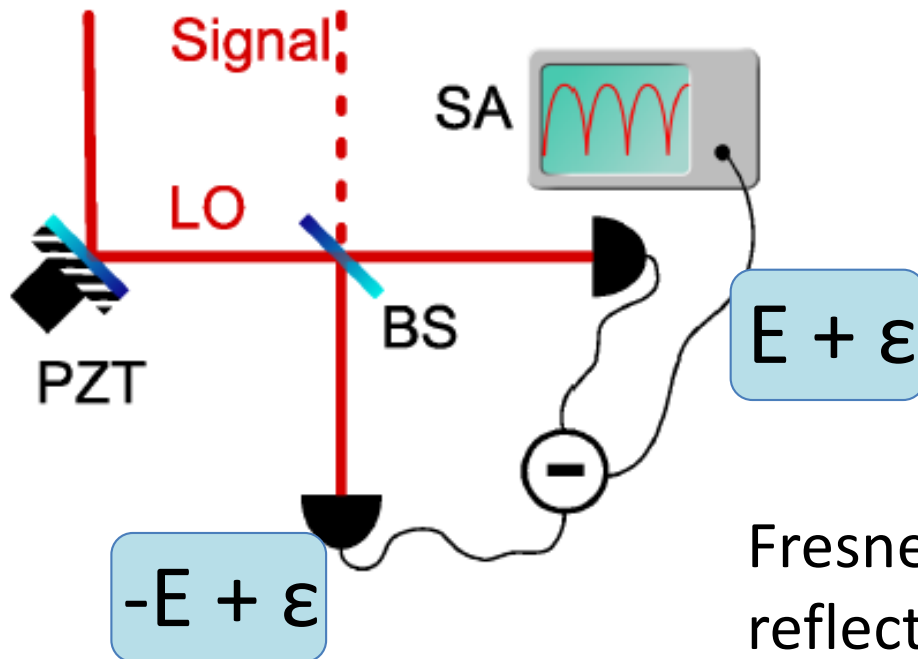
The way we do this is to interfere it with a field with a known phase (a local oscillator - LO).

Homodyning also allows us to amplify and filter as well

Homodyne implies the LO is the same frequency as the signal, while heterodyne detection has the LO and signal at different frequencies.



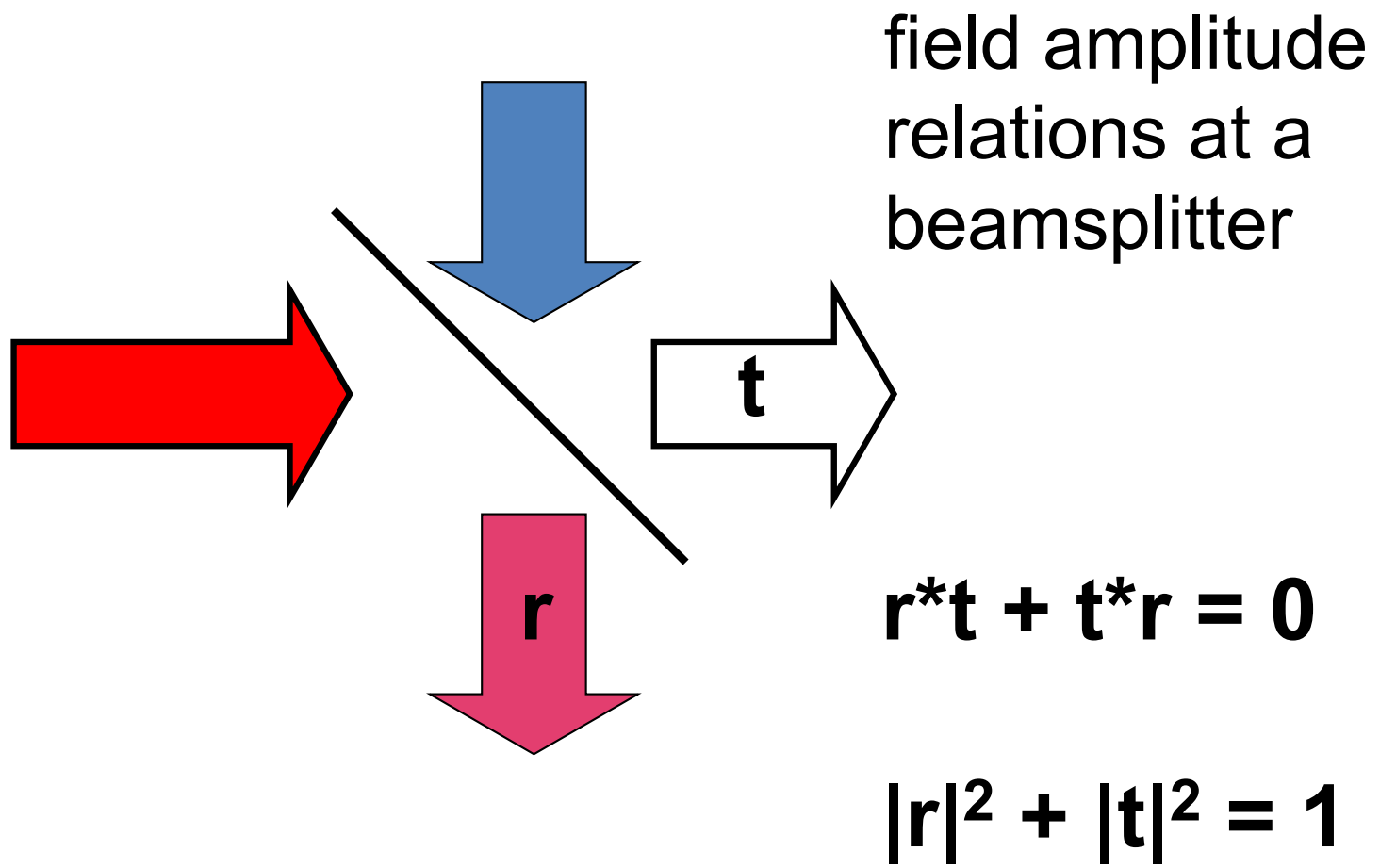
# homodyning light



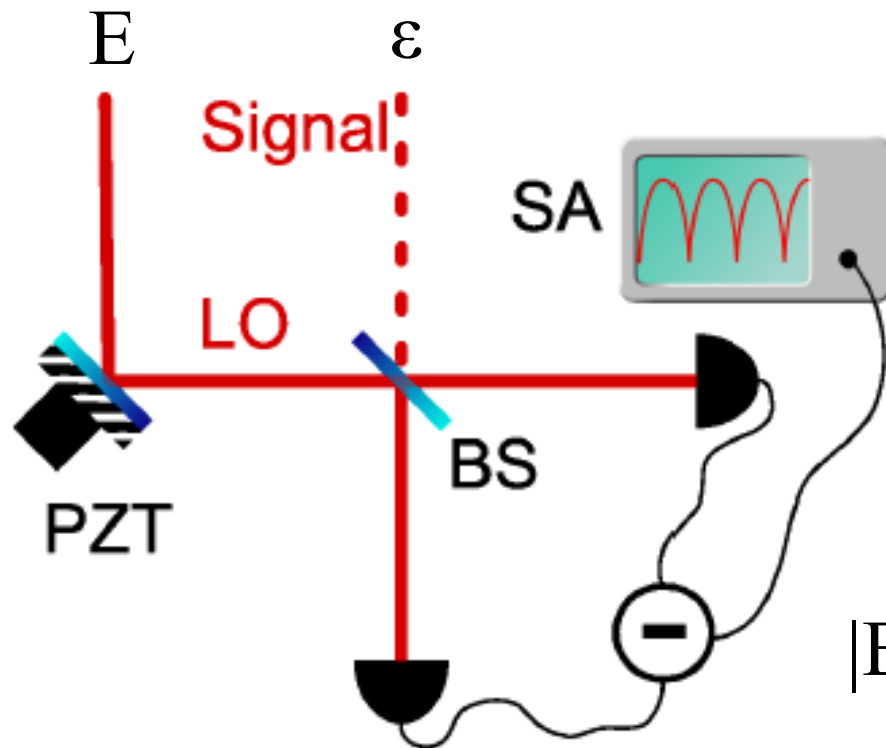
Fresnel equations for beamsplitter reflectivities tells you that one of the reflections must get a minus sign...

$$E(t) \propto X_1 \cos(\omega t) + X_2 \sin(\omega t)$$

# beamsplitter magic



# homodyne detection



$$E \gg \epsilon$$

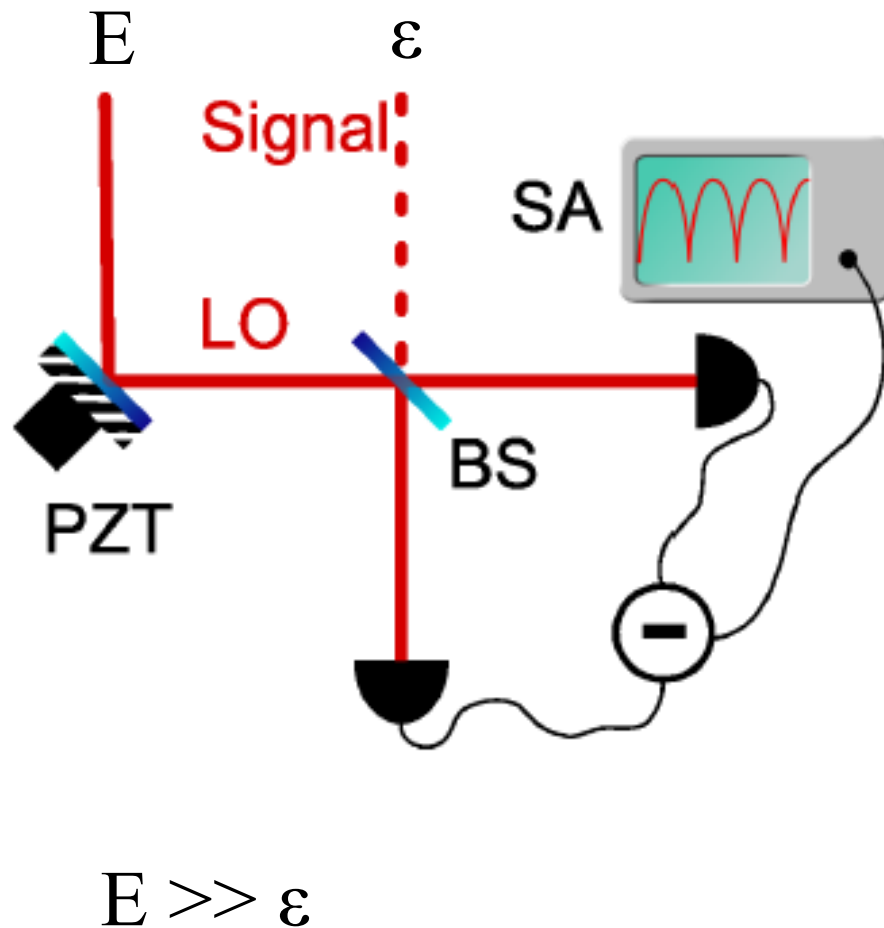
- mix signal with bright beam of same frequency
- get amplification of a small signal
- local oscillator fluctuations cancel out
- phase sensitive

$$|E + \epsilon|^2 = |E|^2 + |\epsilon|^2 + E^* \epsilon + E \epsilon^*$$

$$|-E + \epsilon|^2 = |E|^2 + |\epsilon|^2 - E^* \epsilon - E \epsilon^*$$

$$\text{subtract: signal} = 2\text{Re}(E^* \epsilon)$$

# homodyne (cont.)

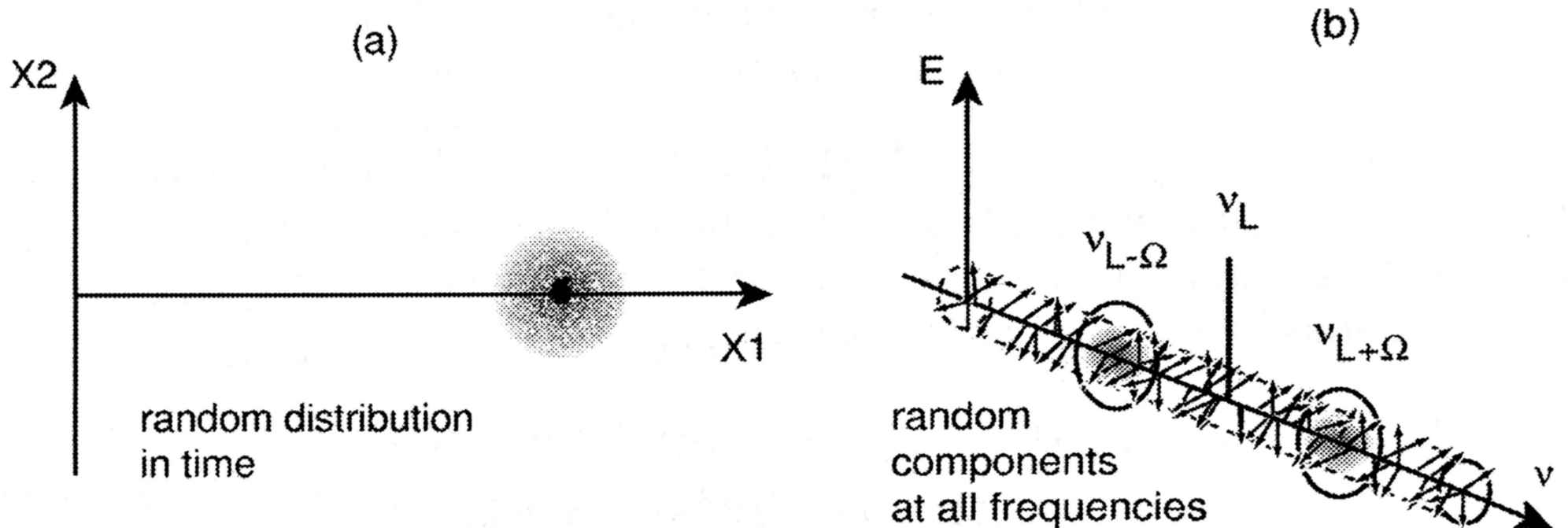


$$\text{signal} = 2\text{Re}(E^*\varepsilon)$$
$$= 2 |E \varepsilon| \sin(\varphi_{LO} + \varphi_s)$$

Thus, one obtains a signal that is:

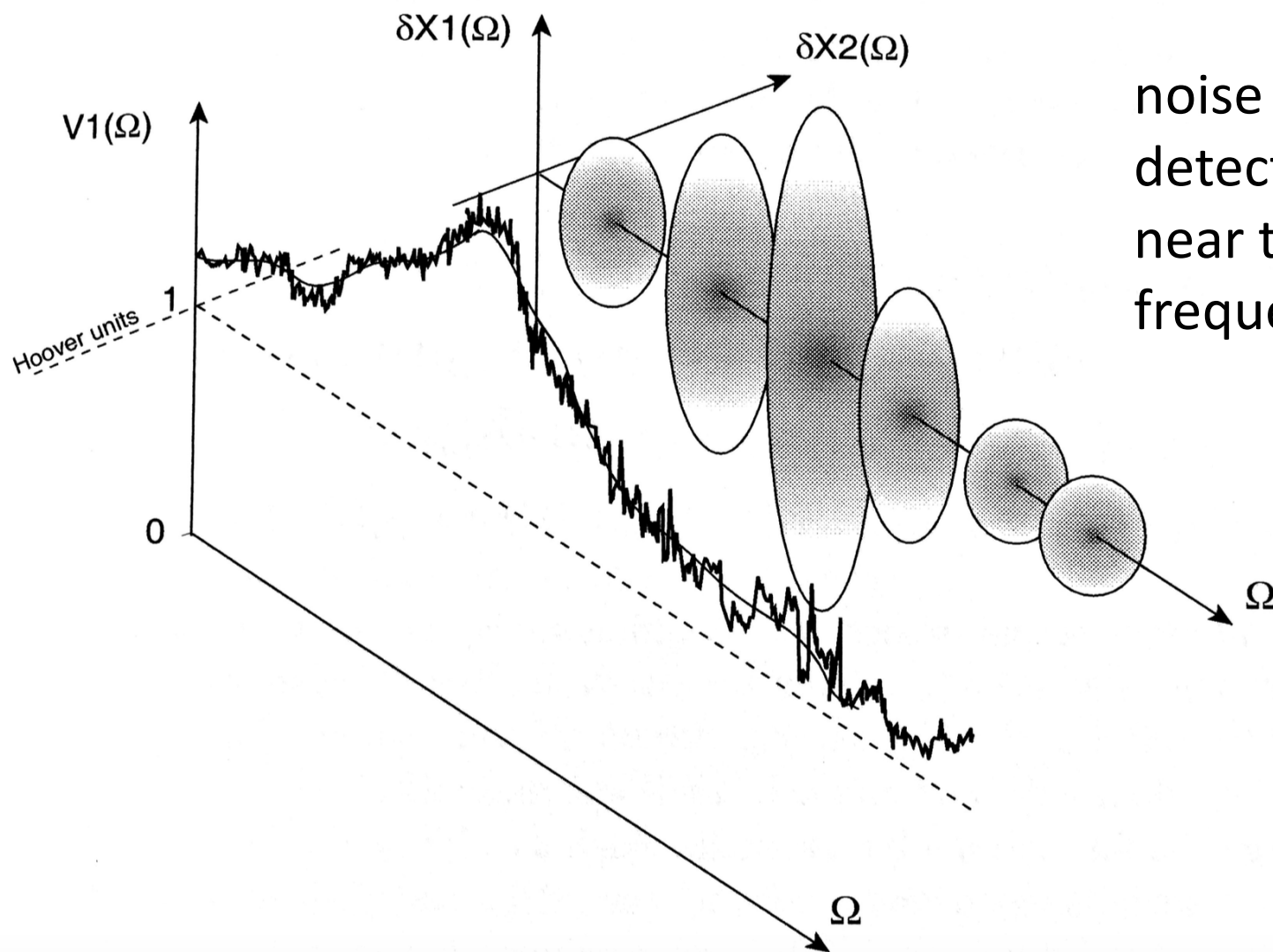
- **proportional to** the weak signal **field** (with phase)
- **amplified** by the strong local oscillator
- strongly **filtered** to the local oscillator frequency bandwidth

# sideband picture for light



figures from "A guide to Experiments in Quantum Optics" by H. Bachor and T. Ralph

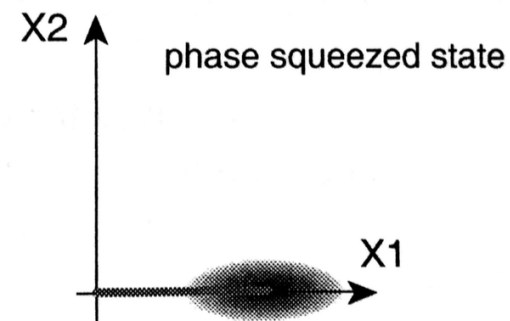
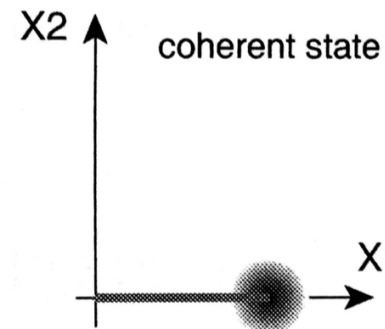
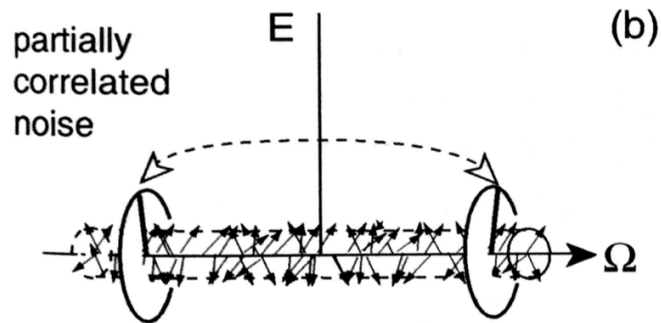
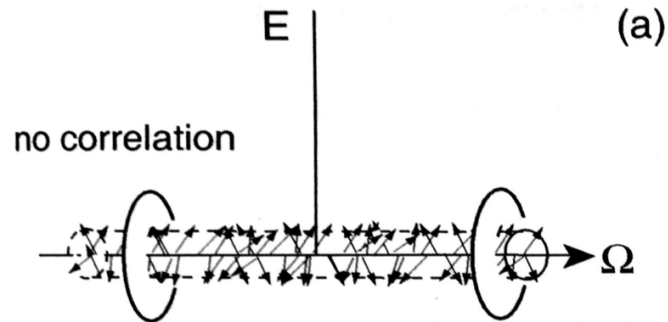
# finite bandwidth signal



noise as a function of  
detection frequency  
near the center  
frequency of a signal

figures from "A guide to Experiments in Quantum Optics" by H. Bachor and T. Ralph

# squeezing in this picture

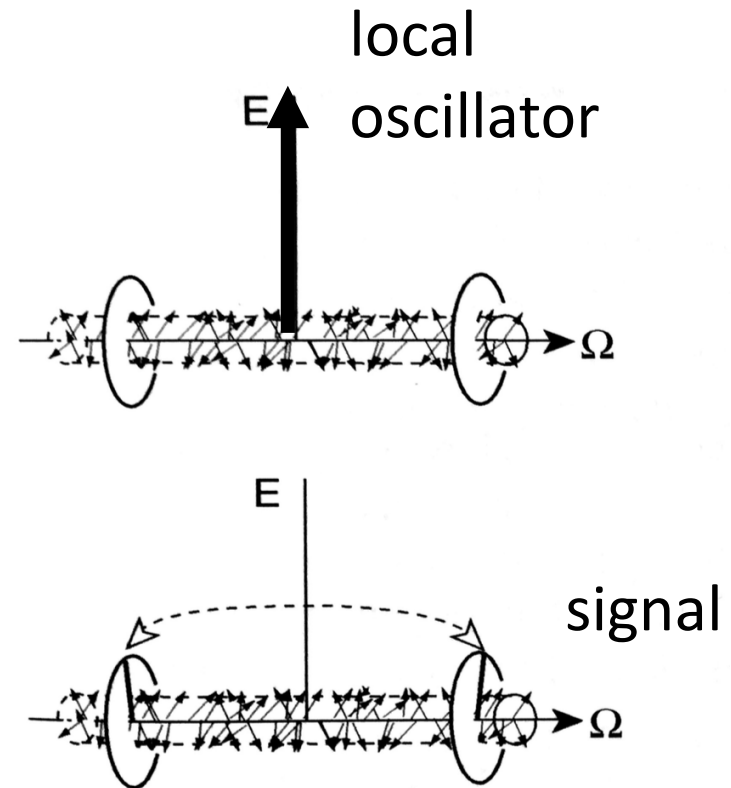
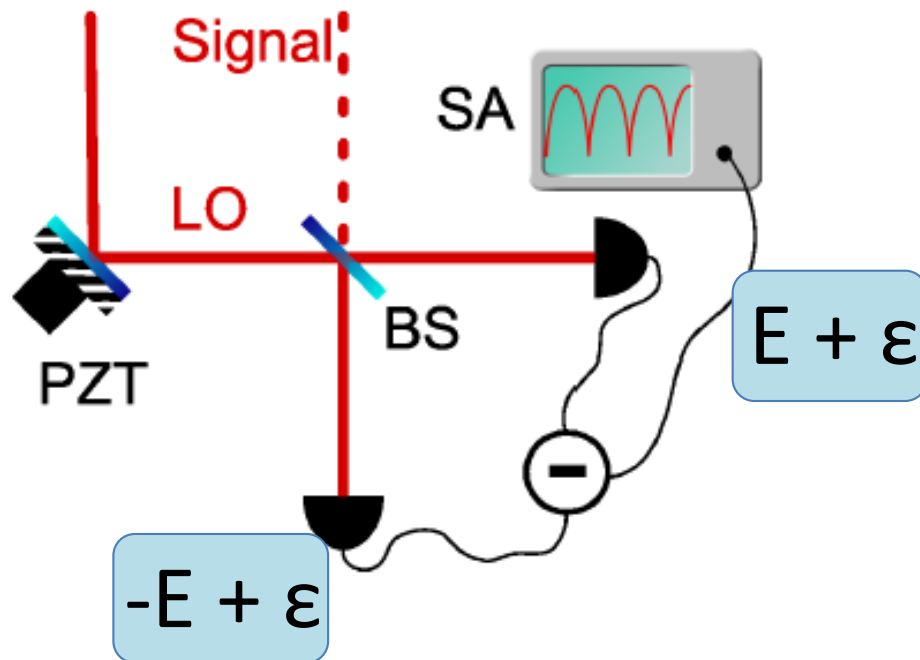


these pictures  
are for a given  
center  
frequency and  
sideband  
frequency

correlations in the sidebands are what result in “squeezing”; the relative phases of the sidebands determine if it is phase or intensity squeezing, etc.

figures from “A guide to Experiments in Quantum Optics” by H. Bachor and T. Ralph

# homodyning light



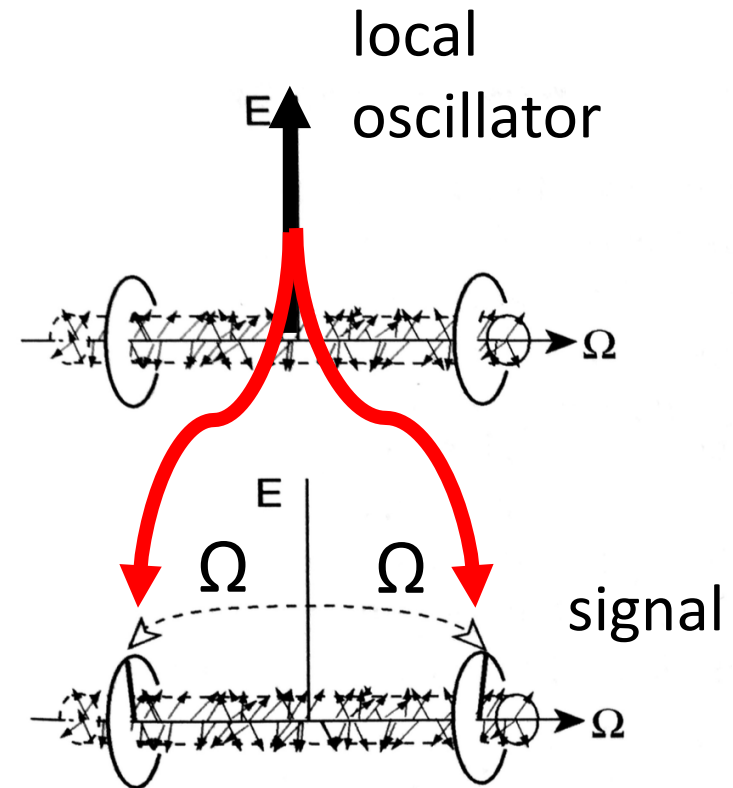
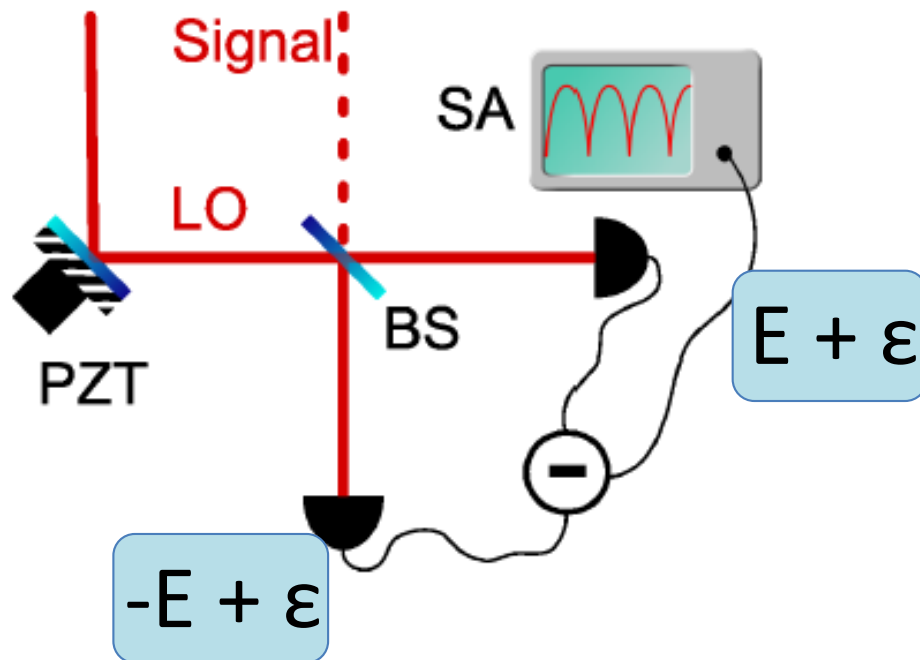
The main signal that we see coming out of the radio frequency spectrum analyzer (SA) at a given frequency is the carrier from the LO beating against the (correlated) signal sidebands.

The LO phase determines which signal quadrature we see.

figures on right from "A guide to Experiments in Quantum Optics" by H. Bachor and T. Ralph



# homodyning light

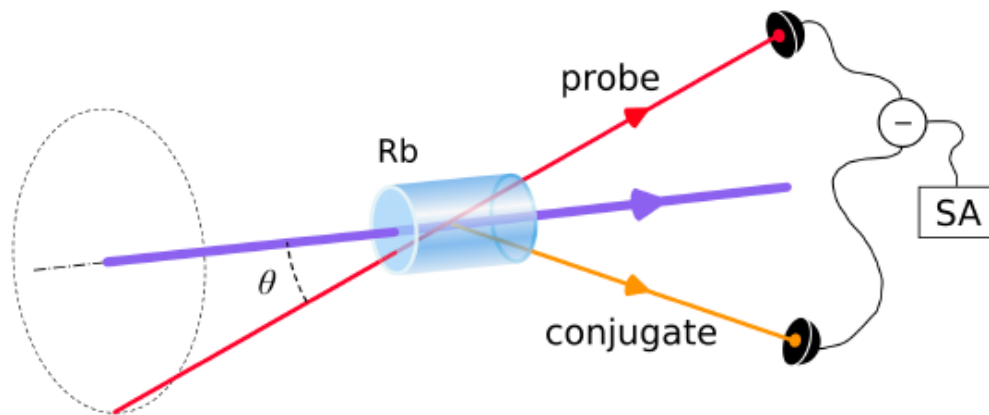


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figures on right from "A guide to Experiments in Quantum Optics" by H. Bachor and T. Ralph

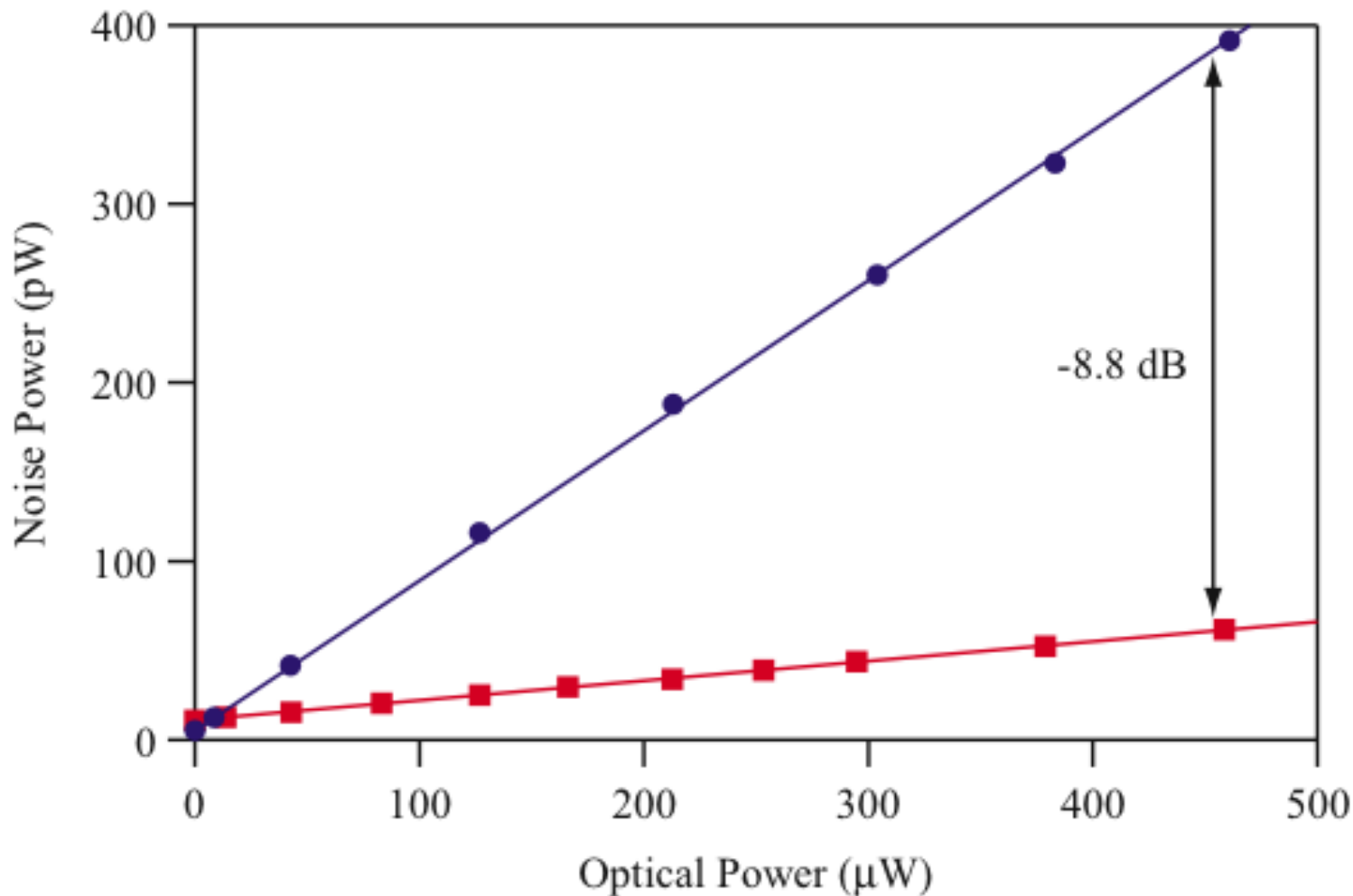
# intensity-difference squeezing



noise power  
measured with  
rf spectrum  
analyzer

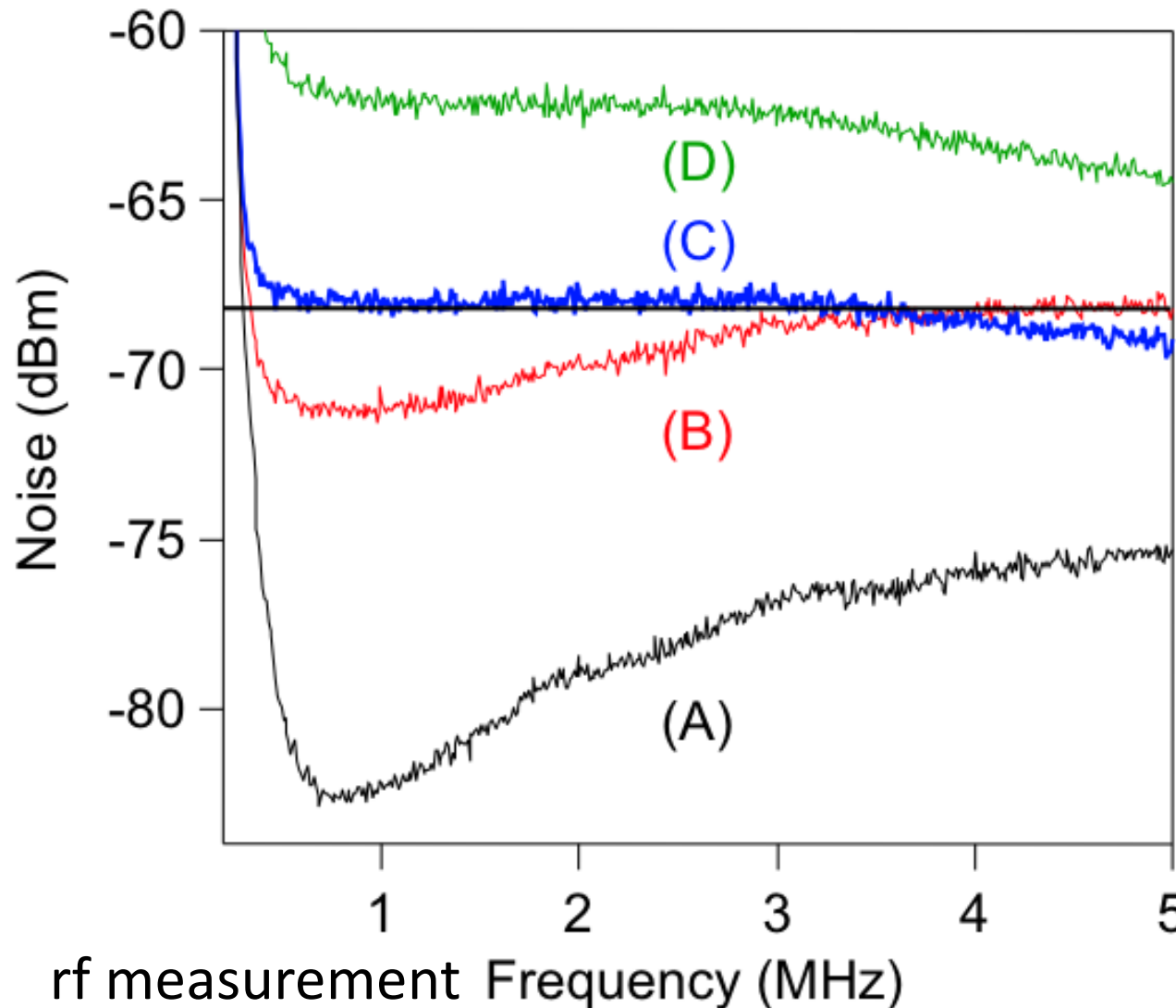
Taking the intensity signals and directly subtracting them (without homodyne detection) is essentially equivalent to looking at only the amplitude (or intensity) quadrature .

# intensity-difference squeezing at a single frequency



1 MHz detection frequency  
RBW 30 kHz  
VBW 300 Hz  
pump detuning 800 MHz  
Raman detuning 4 MHz  
~95% detector efficiency

# spectrum of (intensity-difference) squeezing



squeezing spectrum  
taken with amplified  
detectors with roll-off  
becoming apparent at  
 $\sim 3$  MHz

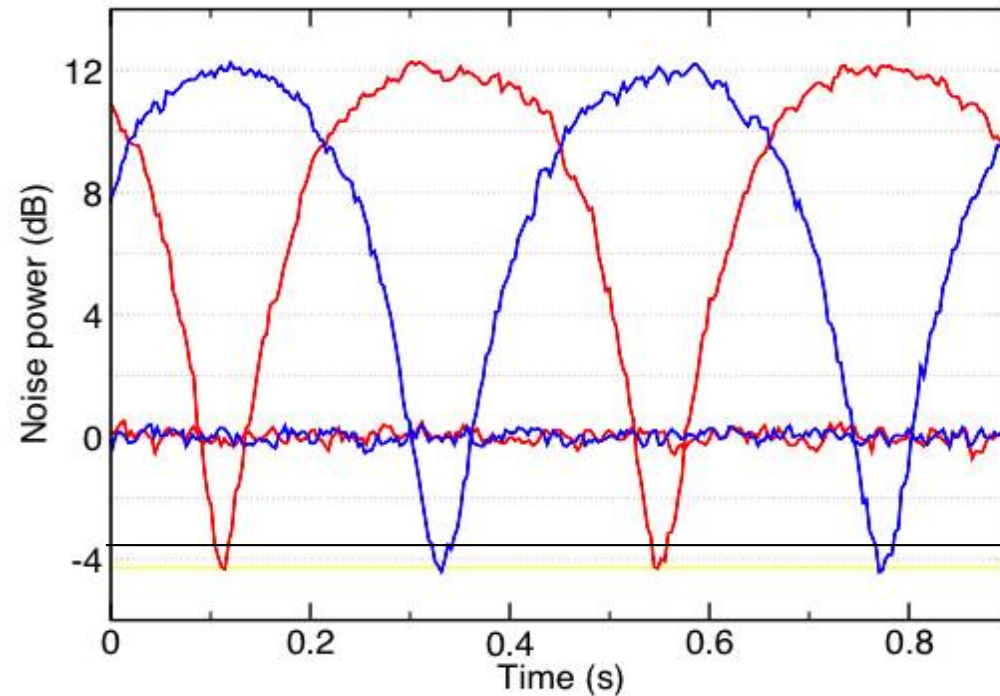
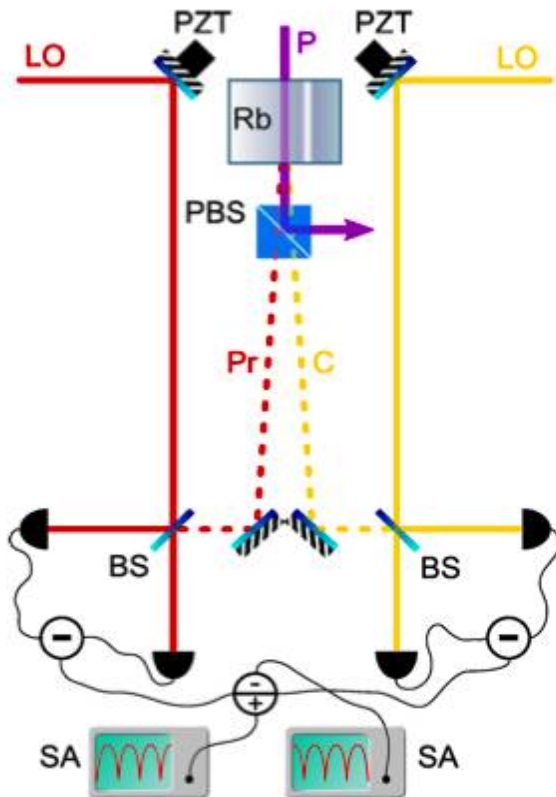
A) electronic noise  
floor

B) relative intensity  
squeezed light

C) SQL for same  
intensity

D) noise from a single  
beam (on 1 detector)  
at same intensity

# “twin beam” vacuum quadrature entanglement



measurements at 0.5 MHz

- Piezos are scanned simultaneously
- Local oscillators are created by 4WM

enough for one day...