Lecture 3

applications of twin beams:

PIA’s (Phase Insensitive Amplifiers)  
PSA’s (Phase Sensitive Amplifiers)  
delayed correlations  
Quantum-correlated images
4WM as an amplifier

• what is the difference between a phase sensitive and a phase insensitive amplifier?
• why is a phase sensitive amplifier valuable?
Quantum Advantages?

• using “squeezed light”

• Note that there have not been an incredible number of applications that have rolled out over the years...

• Generally it has to:

  (1) be worth the substantial hassle, and

  (2) not just be easier to turn up the power.

➤ You are left with light-starved applications or places where you just can’t or don’t want to turn up the power for fear of burning things...
motivational stuff

lets face it... “quantum” generally means “fragile” as well - where is this potentially useful?

soft tissue imaging or imaging live cells or other bits where you can’t turn up the laser power

BEC imaging?

LIGO
Noiseless amplification

sounds too good to be true; like something that must be forbidden...!
There is a “no-cloning theorem” which might say that ...

...but if you don’t want “all the information”... maybe something can be done.

Images typically contain amplitude, but not phase information. If you are happy to amplify the intensity and throw extra noise into the phase, you can do it.
Phase-sensitive amplifiers have been built and can perform noiseless image amplification!

Choi, S.-K., M. Vasilyev, and P. Kumar, Phys. Rev. Lett. 83, 1938 (1999);
PIAs and PSAs

Phase insensitive amplifiers
- a signal that is input is amplified at the output but the signal-to-noise ratio is degraded

Phase sensitive amplifiers
- a signal that is input is amplified or deamplified, depending on the input phase but one can attain noiseless amplification!
What is a PSA?

The noise and/or gain properties are a function of the phase of the input (with respect to the pump fields).

A phase-sensitive amplifier (PSA) is a different type of amplifier:

at the cost of phase sensitivity you can perform noiseless amplification of a (one quadrature) signal!

We can build either a PIA or a PSA with 4WM...
Phase-sensitive amplifier

The phase of the injected beam, with respect to those of the pumps, will determine whether the beam will be amplified or de-amplified.

One can design an amplifier for a given field quadrature.

\[ \omega_+ - \omega_0 = 2 \phi_0 - \phi_- \]

\[ \omega_- + \omega_+ = 2 \phi_0 - \phi_- - \phi_+ \]

given the phase of 3 “input” beams the 4th phase is free to adjust for gain

phase-insensitive

phase-sensitive no free parameters

gain: \[ 0 = 2 \phi_0 - \phi_- - \phi_+ \]
noise limits

phase insensitive amplifier

PIA: \[ NF = \frac{SNR_{in}}{SNR_{out}} = \frac{2G-1}{G} \rightarrow 2 \]

large G limit

phase sensitive amplifier

PSA: \[ NF = 1 \]

(in the ideal case, for the correct choice of signal phase)
Phase-insensitive amplifier

- phase-insensitive gain adds noise; 3 dB “noise penalty” at high gain
- 1 MHz signal amplified with moderate gain
- noise figure $(SNR_{\text{out}}/SNR_{\text{in}}) < 1$ is close to theoretical limit

Phase-insensitive amplifier

- 1 MHz **classical** signal amplified with moderate gain
- noise figure \(\frac{\text{SNR}_{\text{out}}}{\text{SNR}_{\text{in}}}<1\) is close to theoretical limit

squeezing from 4WM in hot Rb vapor

$^{85}$Rb in a double-$\Lambda$ scheme
~1 GHz detuned
~400 mW pump
~100 $\mu$W probe
narrowband
no cavity
non-classical signal amplification

4WM to generate non-classical twin beams

4WM to amplify one of the twin beams
entanglement preserved after amplification

signals measured at 1 MHz
entanglement preservation vs. gain

I = \text{var}(X_\pm) + \text{var}(Y_\pm)
E_{12} = \text{var}(X_1 | X_2) \text{var}(Y_1 | Y_2)

EPR parameter and inseparability
pump schemes

(a) pump schemes

(b) pump schemes
phase sensitive 4WM

given the phase of 3 “input” beams the 4th phase is free to adjust for gain

gain condition:

\[ 0 = 2\phi_0 - \phi_- - \phi_+ \]

different detuning conditions required to suppress other processes as much as possible
Our 4WM PSA

pumps detuned either side of atomic resonance and the relative phase of the beams is controlled
PSA: Quadrature squeezing

- de-amplification phase produces intensity squeezing
- 3 dB single-mode vacuum quadrature squeezing
- multiple spatial modes
- type of squeezing that is injected into interferometers

PSA quadrature squeezing (cont.)

(a) deamplified (bright) signal

predicted squeezing

(b) vacuum squeezing

![Graph showing squeezing vs. One-Photon Detuning Δ [GHz]]
evidence of multi-spatial-mode vacuum quadrature squeezing
PSA noise figure measurement

- phase-sensitive amplifier can provide “noiseless” amplification
  \[ \text{NF} = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} \]
  (inverse of what we used before)
- modest gains, but multi-spatial-mode and nearly “noiseless”

EU PHASORS program has demonstrated higher gain single-mode, fiber-based PSA
PSA noise figure (cont.)

Gain

PIA limit
PSA
PSA w/ loss

\( \Delta \text{One-Photon Detuning [GHz]} \)
Seeded 4-wave mixing

seeded, bright modes

conjugate

probe seed

$^{85}\text{Rb}$

pump

mask

cone of vacuum-squeezed modes
(allowed by phase matching)

$\theta$

$\Delta \theta$
no cavity, so freedom for complex and multiple spatial modes!
multi-spatial-mode noiseless (image) amplification

look at the change in noise figure with the attenuation/“spatial trimming” of the image
demonstrating entanglement

scan LO phase

alignment and bright beam entanglement

probe

conjugate

phase stable local oscillators at +/- 3GHz from the pump
demonstrating entanglement

vacuum squeezing

unsqueezed vacuum

probe

conjugate

pumps

50/50 BS

signal pump

LO pump

pzt mirror

scan LO phase

pzt mirror

+ and -
“twin beam” vacuum quadrature entanglement

- Piezos are scanned simultaneously
- Local oscillators are created by 4WM

measurements at 0.5 MHz
Entanglement and EPR violations with images

Vacuum squeezing with Gaussian-shaped local oscillators

Vacuum squeezing with T-shaped local oscillators

EPR violation

\[ E_{12} - \text{Var}(X_1|X_2)\text{var}(Y_1|Y_2) < 1 \]

violation guaranteed with 3dB squeezing in each quadrature

measurements at 0.5 MHz
Seeded 4-wave mixing

seeded, bright modes

conjugate

probe seed

\(^{85}\text{Rb}\)

pump

mask

LO beam misalignment

missing some part of mode

homodyning some part of uncorrelated squeezed vacuum

cone of vacuum-squeezed modes (allowed by phase matching)

\(\theta\)

\(\Delta \theta\)
squeezed and entangled cats

local oscillators for measurements of 1 dB quadrature squeezed vacuum

bright beams showing intensity-difference squeezing

~1 dB “whole image” intensity-difference squeezing
multi-spatial-mode squeezing

- short cell relaxes phase-matching conditions
- diffraction limit of pump spot in cell sets the size of the coherence area
- approximately 100 independent modes
advancing or delaying signals

“Slow light” and “fast light,” or group velocities for pulses or fluctuations that are very different from $c$, can be used to manipulate signals.

One wants the ability to advance or delay signals to pack communications signal packets into a channel. It would also be useful for quantum communications technologies to have this ability to manipulate quantum information.

How does such manipulation affect quantum information? What are the limits of what we can do?
In particular, if we try to advance a signal (quantum or otherwise) isn’t there a problem that we will have sending signals faster-than-light?

What can we do to investigate this?

We will investigate the propagation of information in the next lecture but for now we can look at the generation of slow and fast light in these systems and how well we can preserve entanglement (a type of information) in the delay of images...
Slow Light

EIT (less loss) spectrum with associated dispersion

group velocity \( v_g = \frac{c}{n - \lambda_0 \frac{dn}{d\lambda_0}} \)

gain features imply dispersion and slow light as well

squeezing or entanglement delay

~10 ns delays  ~30 ns extra delay

2nd cell gain adjusted to be ~1 with temperature and pump intensity
Entanglement Delay

- Delay can be tuned by changing the gain:
  - Temperature
  - Pump power

- Fractional delay:
  \[
  \frac{\text{delay}}{\text{width } g^{(2)}}
  \]

\[EPR \Rightarrow 0.5\]
\[\mathcal{I} \Rightarrow 0.65\]
(without 2\textsuperscript{nd} cell)
Generation of conjugate in second cell (due to gain) degrades the entanglement.

\[ E_{12} = \text{var}(X_1|X_2)\text{var}(Y_1|Y_2) \]

\[ I = \text{var}(X_-) + \text{var}(Y_+) \]

- Effect of gain on entanglement

Delay of Entangled Images

- Fractional delay of about 1/3
how does delay affect squeezing?

If quantum correlations result in the reduction of noise, won’t moving those correlations around sort of mess that up?

We have talked about this to some extent but we can look at the squeezing spectrum as well...
probe-conjugate delay is important!

There is little delay in the emission of a probe photon following the emission of a conjugate photon; the photons are expected to be highly correlated in time.

The important delay is the propagation delay in the cell!

Slow light and correlations

We can achieve slow light in much the same way as EIT produces the effect: the transparency window/gain feature implies a dispersive feature in the index of refraction.

In our 4WM cell we have two pulses (fluctuations) propagating at slightly different frequencies and see locked pulse propagation as well as the slow light effects.

For given intensity/ detunings, after a “transient regime” we find that a 100 ns pulse input at the probe frequency emerges from the cell with a ~50 ns delay; the conjugate emerges ~10 ns ahead, regardless of cell length (within limits).
correlation delay

If the conjugate photons come out ahead of the probe photons, how does this affect the squeezing?

Look at intensity cross-correlation function to measure differential delay.

If the conjugate photons come out ahead of the probe photons, how does this affect the squeezing?
Controlling squeezing spectrum with conjugate delay line

Optically delay conjugate by variable times
Squeezing spectrum extended to ~ 15 MHz

Controlling squeezing spectrum with conjugate delay line
12-13-2006
P = 1.3 mW, T = 153C
1517 MHz, +0.7 GHz
R/V BW = 30 kHz, 30 Hz
G = 10^4 V/A
Spectra with electronic noise floor subtracted

Controlling squeezing spectrum with conjugate delay line

Background subtracted

12-13-2006

P = 1.3 mW, T = 153°C
1517 MHz, +0.7 GHz
R/V BW = 30 kHz, 30 Hz
G = 10^4 V/A

4WM; no delay

4WM; 6.2 ns

SQL

4WM; 13.5 ns
The spectrum of squeezing is the Fourier transform of time- and normally-ordered correlation function of the intensity difference.

\[ S(\Omega) = \int_{-\infty}^{\infty} d\tau e^{-i\Omega \tau} \langle : z(t)z(t + \tau) : \rangle \]

\[ z(t) = I_1(t) - I_2(t) \]

The spectrum of squeezing is the Fourier transform of time- and normally-ordered correlation function of the intensity difference.

\[ \langle z(t)z(t + \tau) \rangle = A_{I_1}(\tau) + A_{I_2}(\tau) - 2C_{I_1I_2}(\tau) \]

auto- and cross-correlations

\[ S(\Omega) = S_{I_1}(\Omega) + S_{I_2}(\Omega) - 2S_{I_1I_2}(\Omega) \]
Squeezing spectrum with delay

\[ S(\Omega,T) = \int_{-\infty}^{\infty} d\tau e^{-i\Omega\tau} \langle : y(t,T)y(t+\tau,T) : \rangle \]

\[ y(t,T) = I_1(t) - I_2(t+T) \]

\[ \langle y(t,T)y(t+\tau,T) \rangle = A_{I_1}(\tau) + A_{I_2}(\tau) - C_{I_1I_2}(T+\tau) - C_{I_1I_2}(T-\tau) \]

\[ \text{Fourier transform property:} \quad f(t) \Leftrightarrow F(\Omega) \]
\[ f(t-\tau) \Leftrightarrow e^{-i\Omega\tau} F(\Omega) \]

\[ S(\Omega,T) = S_{I_1}(\Omega) + S_{I_2}(\Omega) - S_{I_1I_2}(\Omega)e^{i\Omega T} - S_{I_1I_2}(\Omega)e^{-i\Omega T} \]

\[ S(\Omega,T) = S_{I_1}(\Omega) + S_{I_2}(\Omega) - 2S_{I_1I_2}(\Omega)\cos(\Omega T) \]
Squeezing with a delay

- single beam noise
- default from the 4WM cell
- shot noise limit
- "optimal" delay to remove differential slow-light delay (8.5 ns)

**Explanation:**
Squeezing with more delay (linear scale)

Fourier transform of the intensity sum; delay optimized

Fourier transform of the intensity difference; delay optimized
end of lecture 3

lecture 4
Fisher information and entropy
Mutual information through PIAs and PSAs
matter wave correlations