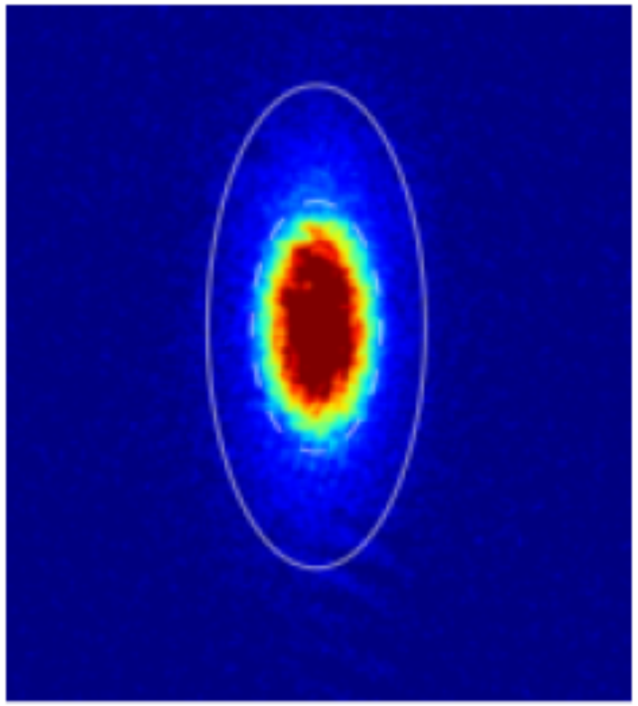
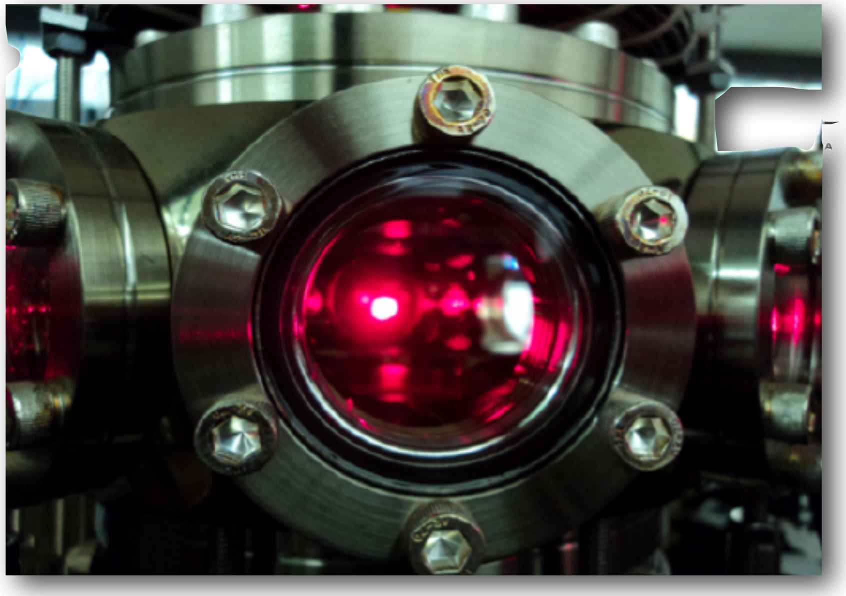
THE ESSENTIAL TOOL: light !!!



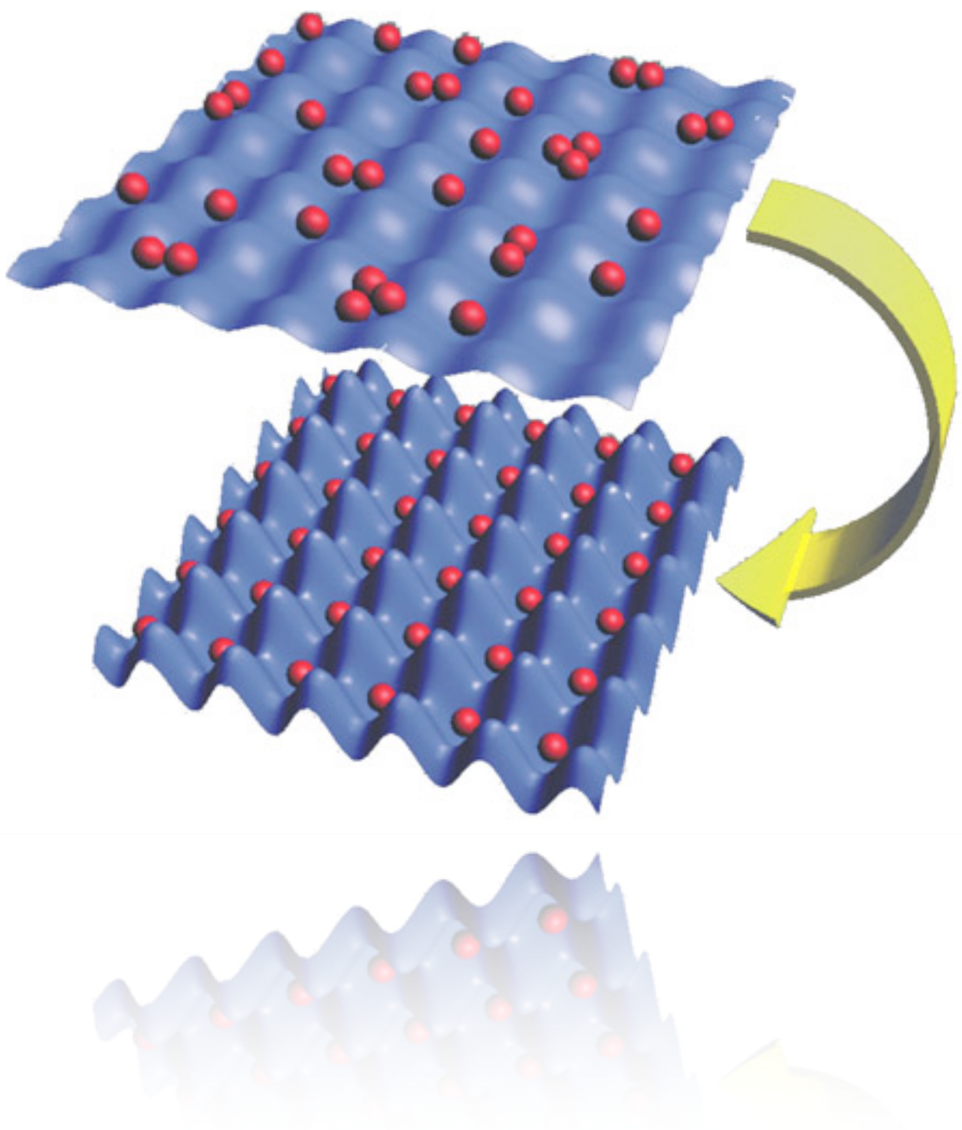
Imaging

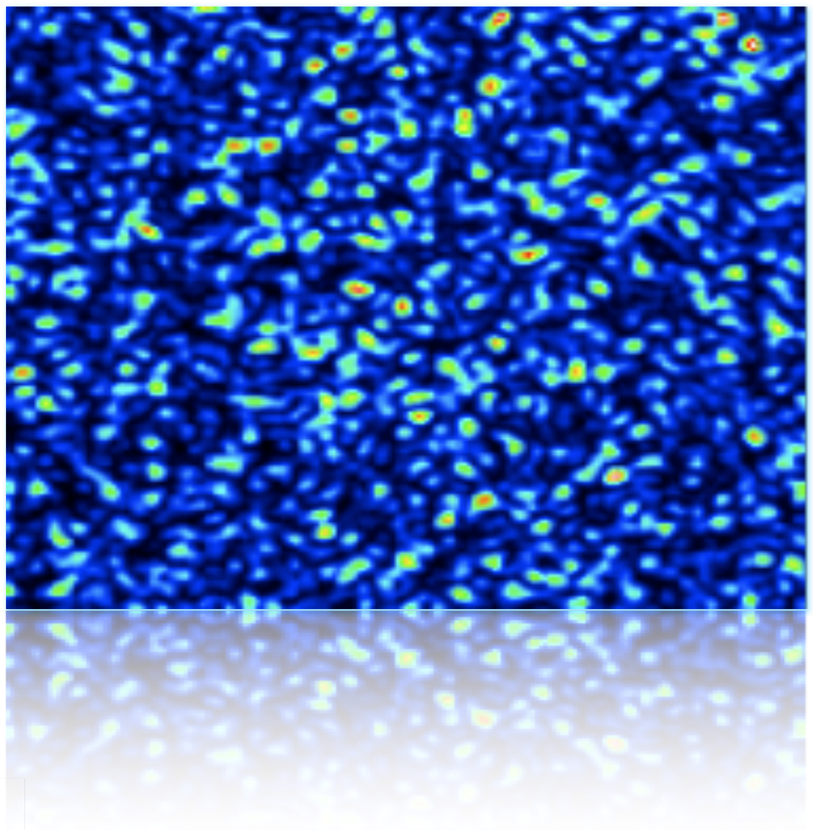
Trapping

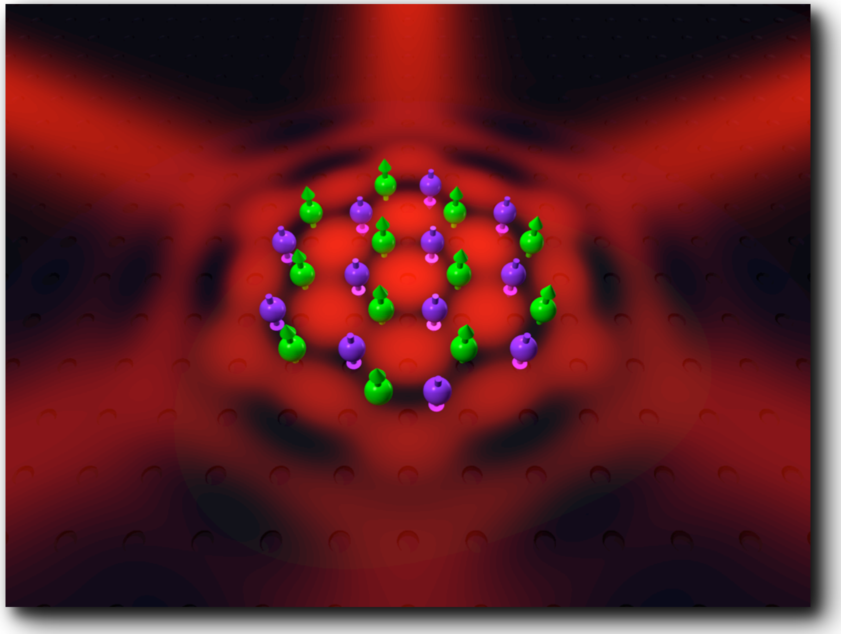


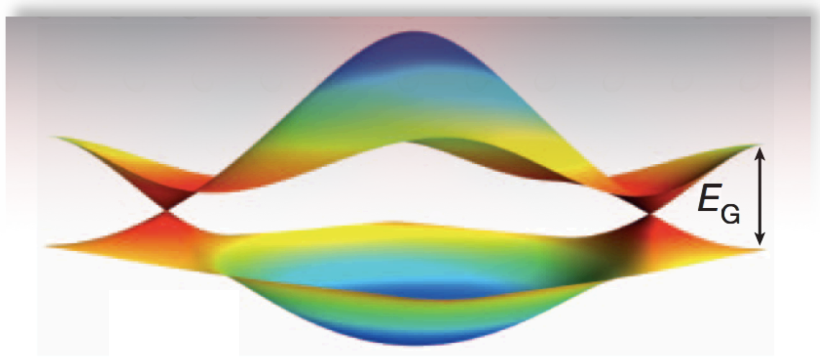
Cooling

Engineering artificial “crystals” made by (laser) light



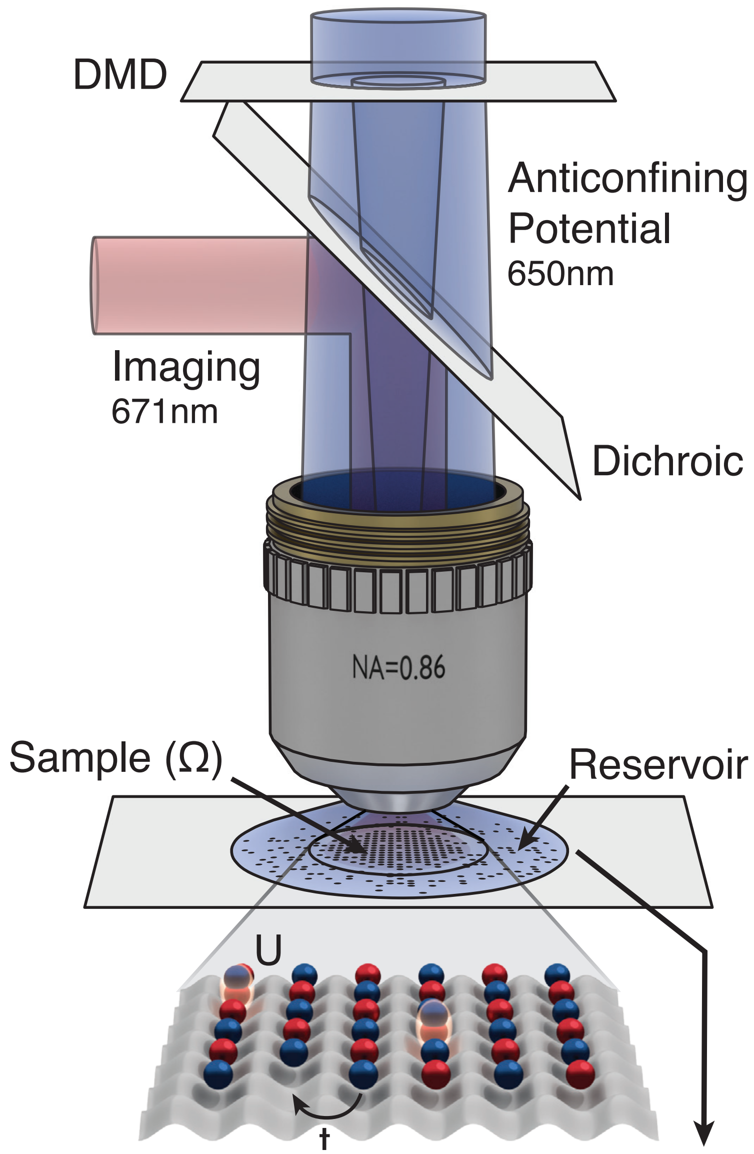


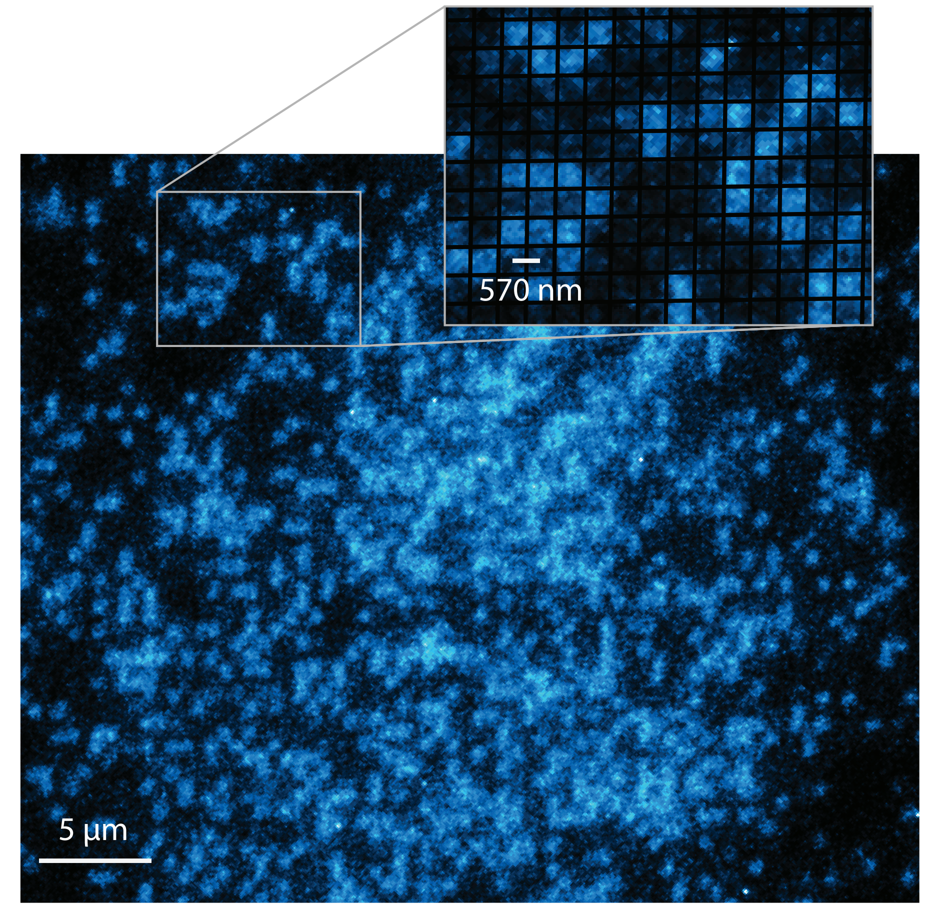


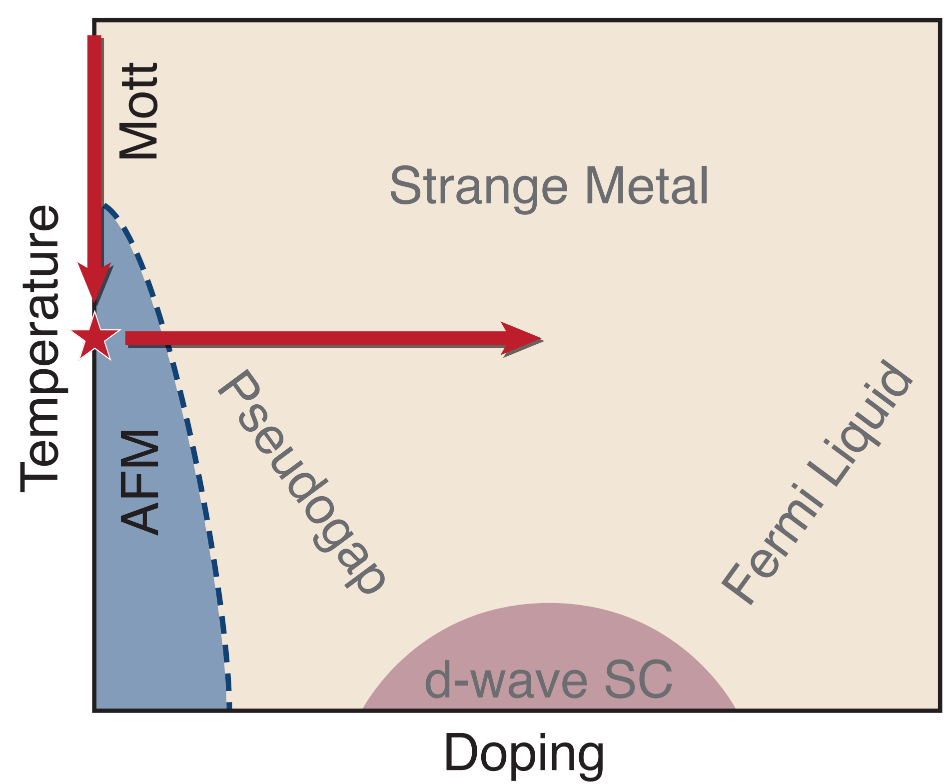


I. Bloch, et al., Rev. Mod. Phys., **80**, 885 (2008)

$$\hat{H} = -J \sum_{\langle \mathbf{n},\mathbf{m} \rangle} \left(\hat{c}^{\dagger}_{\mathbf{n},\sigma} \hat{c}_{\mathbf{n},\sigma} + \mathrm{h.c.} \right) + U \sum_{\mathbf{n}} \hat{n}_{\mathbf{n},\uparrow} \hat{n}_{\mathbf{n},\downarrow} \,,$$

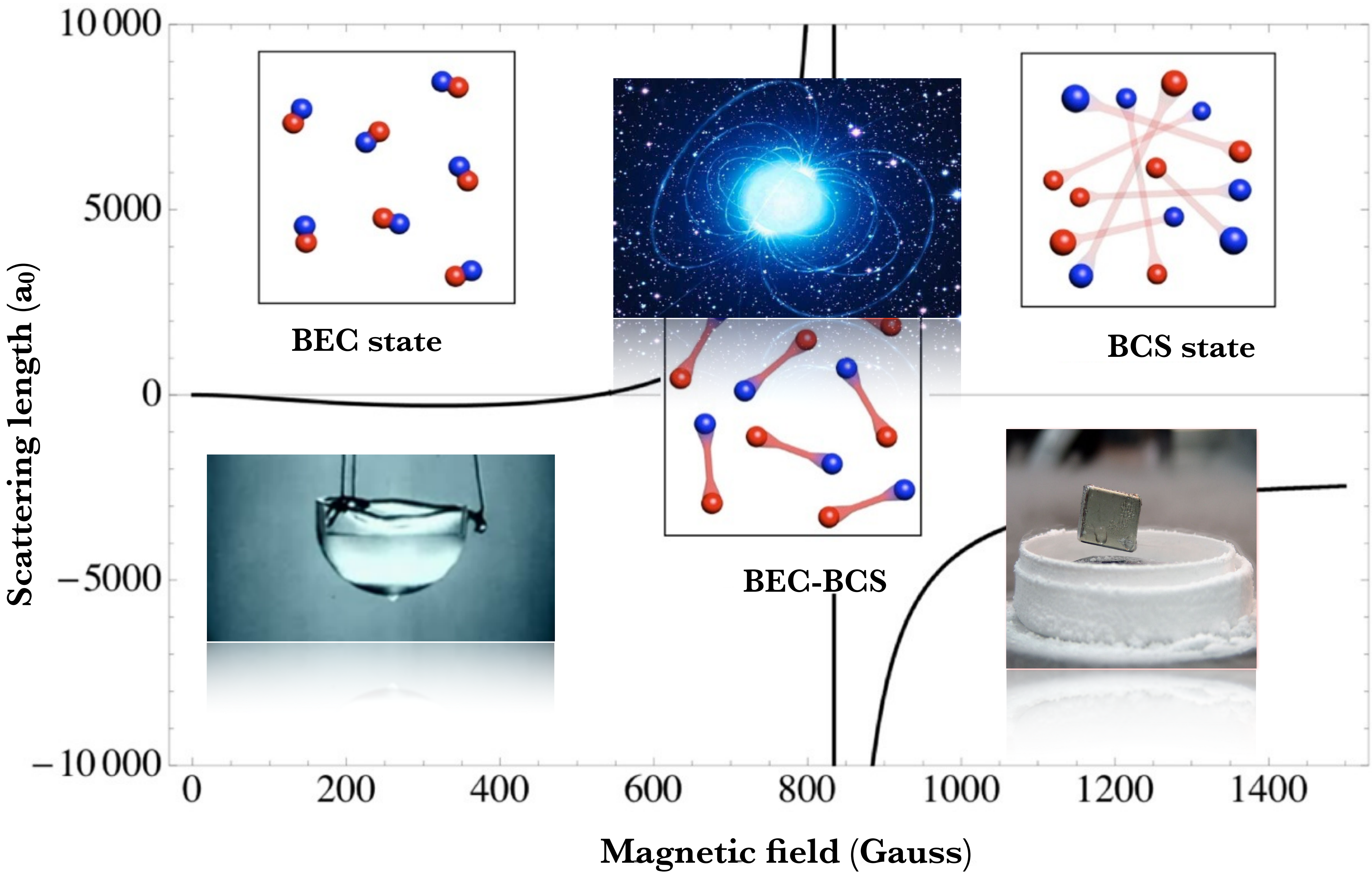






M. Greiner's group

Entering the fermionic land...



Proceedings of the International School of Physics "Enrico Fermi", Course CLXIV, Varenna, edited by M. Inguscio, W. Ketterle, and C. Salomon (IOS Press, Amsterdam)

Bosons

Fermions

$$f_{BE}(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} - 1}$$

$$f_F(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

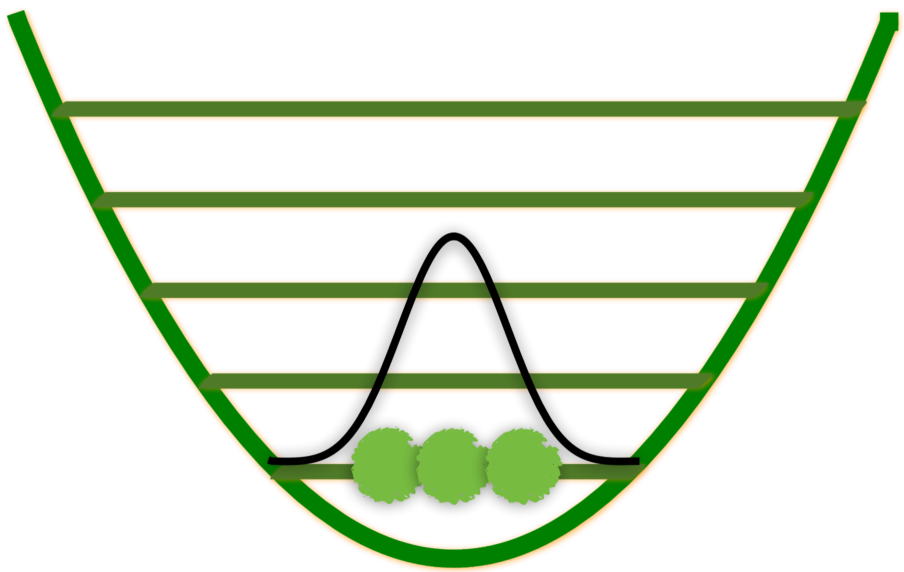
In 1926, when he was professor in Florence, Enrico Fermi published his famous article on the "quantisation of the ideal monoatomic gas", unveiling the behavior of the particles that, after him, would have been named as fermions.

On the Quantization of the Monoatomic Ideal Gas
"Rend. Lincei",3,145-149 (1926)

$$f_F(\epsilon) = \frac{1}{e^{\beta(\epsilon-\mu)} + 1}$$

English version @
arXiv:cond-mat/9912229

Bosons: Bose-Einstein condensate
(T=0)



• Statistical “attraction” between the particles

- Atomic gases
- Photons
- Phonons in crystal
- ^4He

Atomic Fermi gases (in harmonic traps)

The physics depends on T/T_F

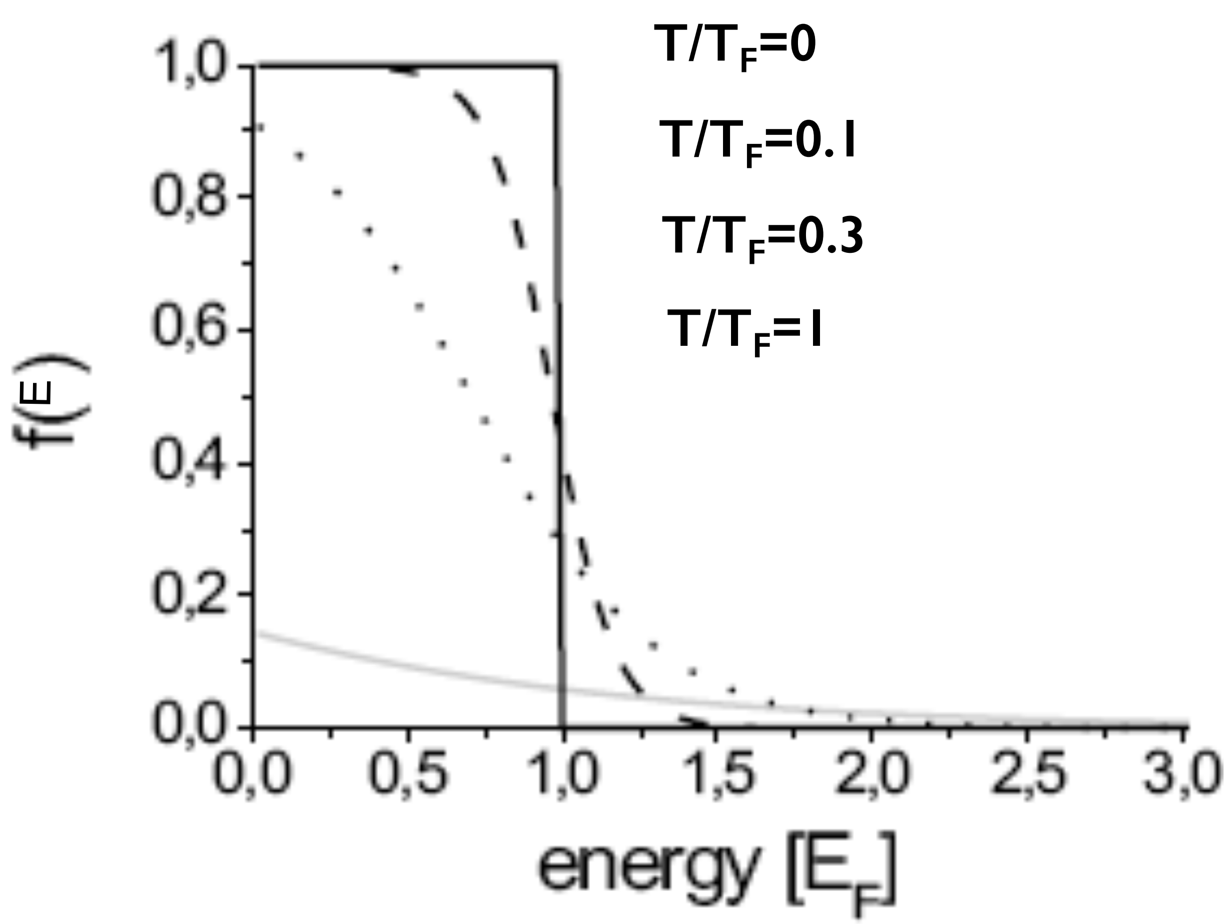
$T/T_F>1$: classical regime -- $T<T_F$: quantum regime

T=0: the Fermi-Dirac distribution is: $$f_F(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}$$

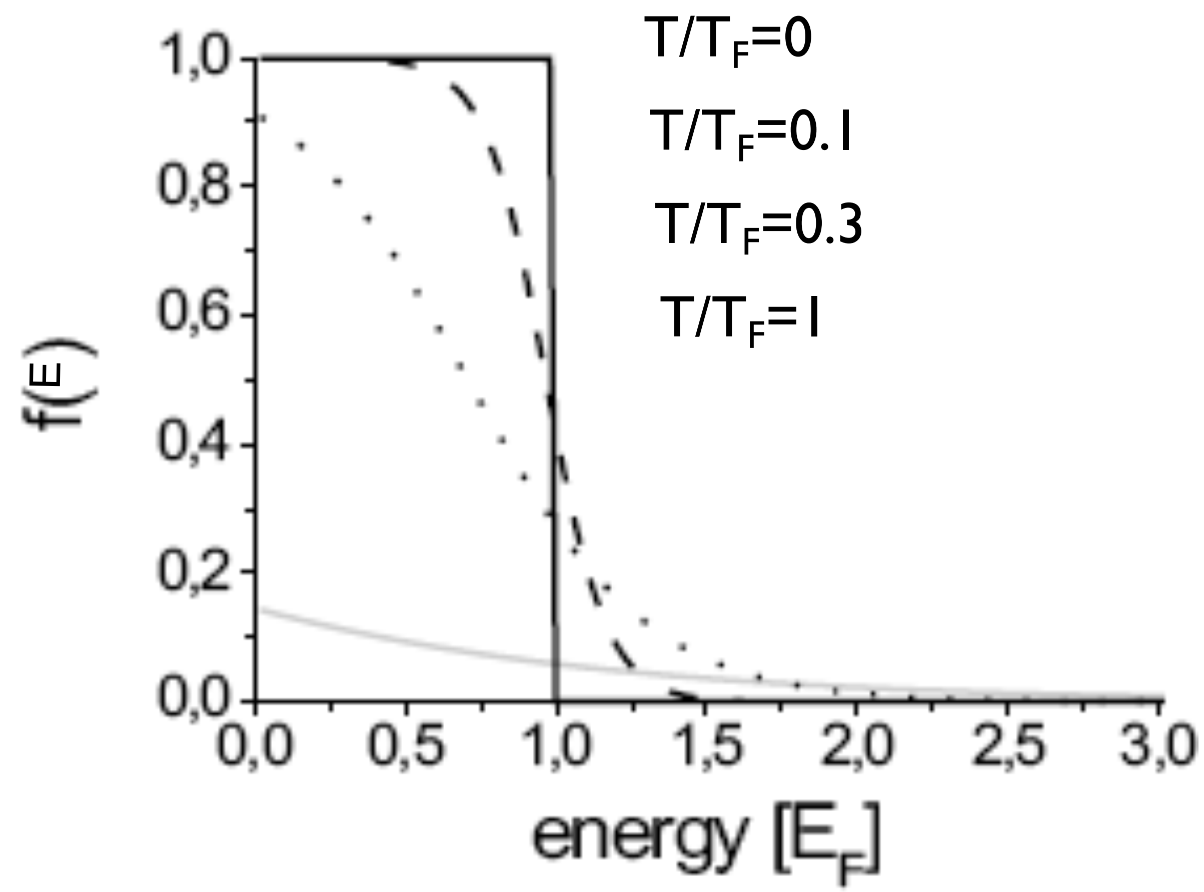
$\beta=1/k_BT$, μ is the chemical potential.

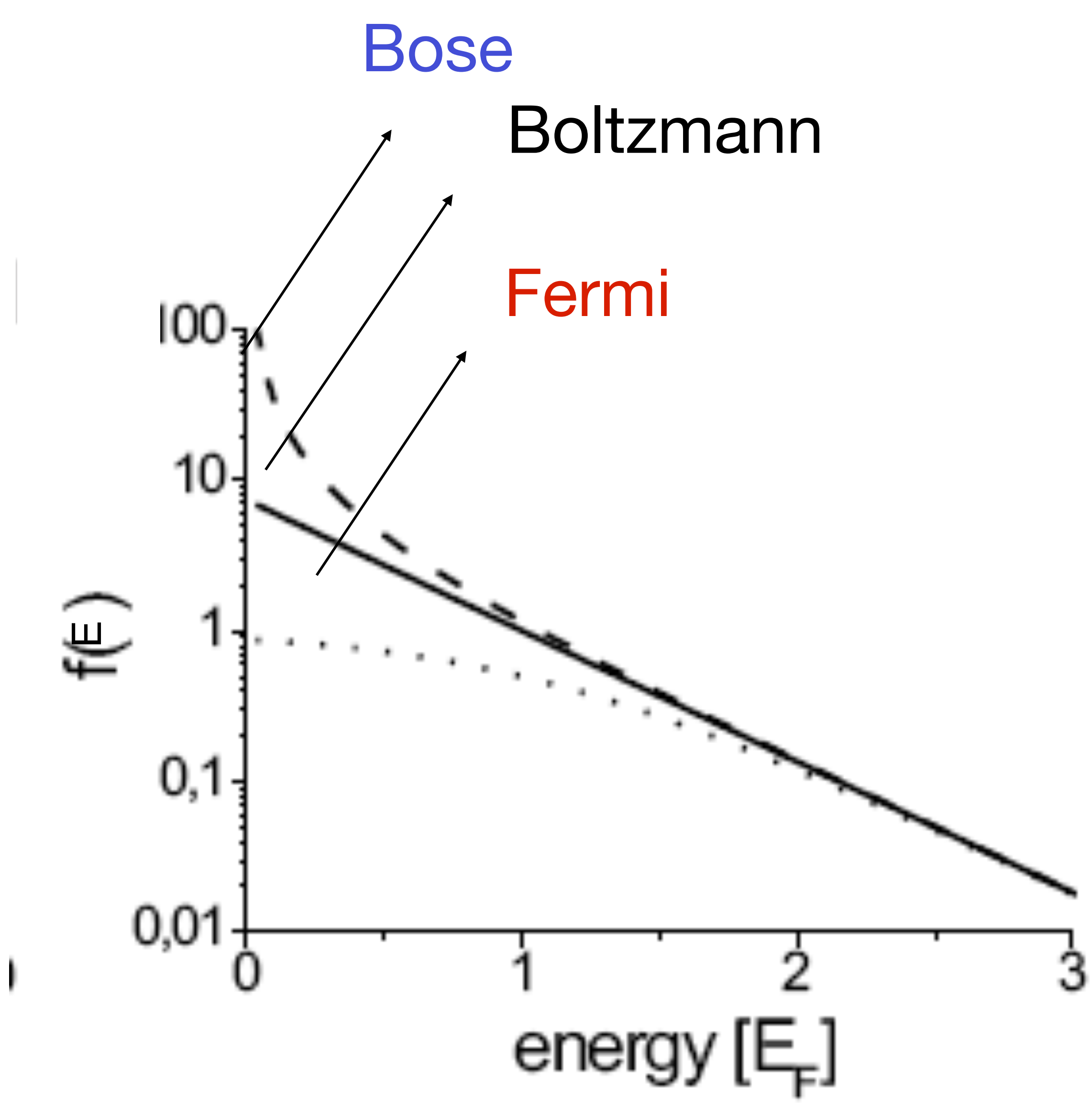
$E_F=\mu$ @ T=0.

The Fermi-Dirac distribution never exceeds 1, reflecting the Pauli exclusion principle



$T/T_F \neq 0$: the step function is smeared out around E_F with a width of the order of $\sim E_F \times T/T_F$.





If $T/T_F > 1$ the Fermi-Dirac becomes the "classical" Boltzmann distribution:

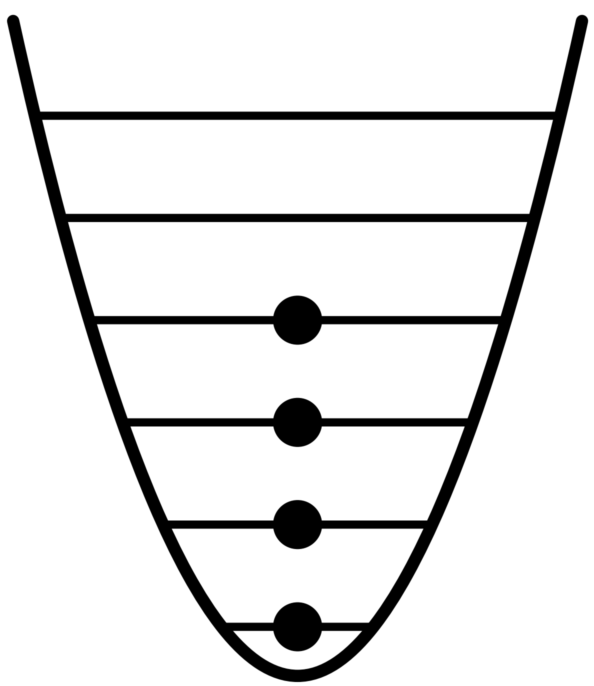
$$f\,(\varepsilon) \approx e^{-\beta(\varepsilon-\mu)}$$

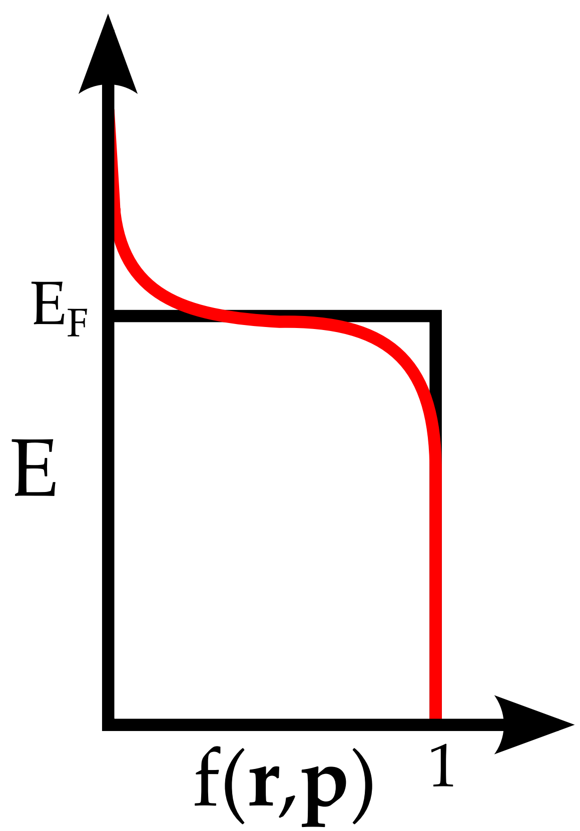
N identical fermions in a cylindrical **harmonic** trap:

$$V(\rho) = \frac{1}{2} m \omega_r^2 \rho^2$$

where $\rho^2=(x^2+y^2+\lambda z^2)$ and $\lambda=\omega_a/\omega_r$ is the anisotropy of the trap

The **Pauli principle** forbids multiple occupation of a single quantum state: fermions occupy one by one every state. The energy of the last occupied state (last fermion) is the Fermi energy E_F. Fermi temperature: $T_F=E_F/k_B$





If $\hbar\omega_r, \hbar\omega_a << k_B T$ we can define the density of state as:

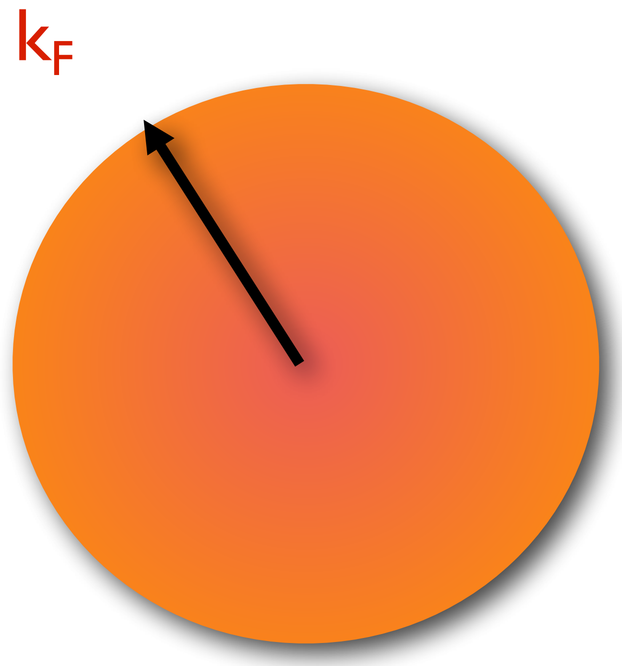
$$g(\varepsilon) = \frac{\varepsilon^2}{2(\hbar\varpi)^3}$$

The total number of atoms of the system is given simply by:

$$N = \int d\varepsilon \frac{g(\varepsilon)}{e^{\beta(\varepsilon-\mu)}+1}$$

from which it is possible to obtain the explicit relation for the Fermi energy:

$$E_F = \hbar\varpi\sqrt[3]{6N}$$



$$k_F = \frac{1}{\hbar}\sqrt{2mE_F}$$

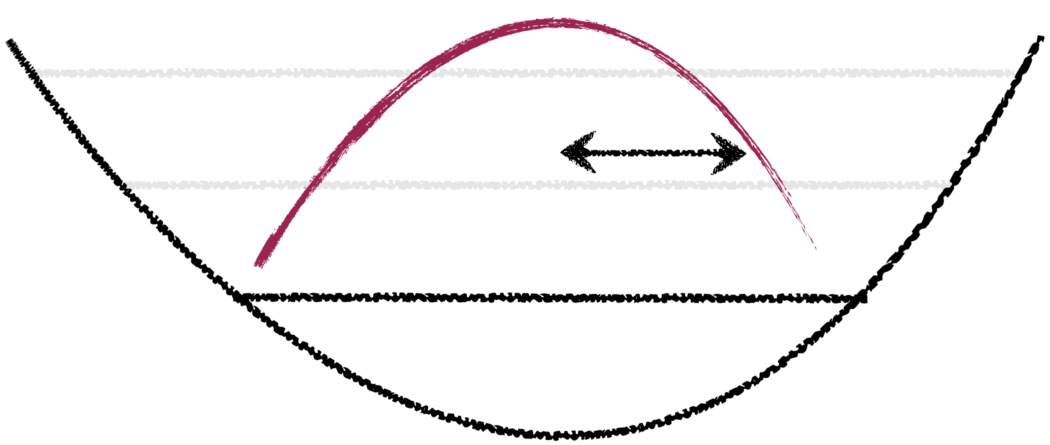
In momentum space, the step function corresponds to a sphere of radius k_F: @ T=0 all the momentum states ≤ k_F are occupied! ➔ consequences on the collisions between fermions (Pauli Blocking)

Another important quantity which can be defined is the Fermi radius of our trapped degenerate gas:

$$R_F = \sqrt{\frac{2E_F}{M\omega_r^2}}$$

$$\boxed{\begin{aligned} E_F &= \hbar\varpi\sqrt[3]{6N} \\ R_F &= \sqrt{\frac{2E_F}{M\omega_r^2}} \end{aligned}}$$

It is worth to compare the size of the Fermi gas with the harmonic oscillator length of the ground state of our trap:



$$a_{ho} = \sqrt{\frac{\hbar}{m\omega}}$$

We directly get: $$R_F \propto a_{ho}\sqrt[6]{N}$$

$$R_F \propto a_{ho}\sqrt[6]{N}$$

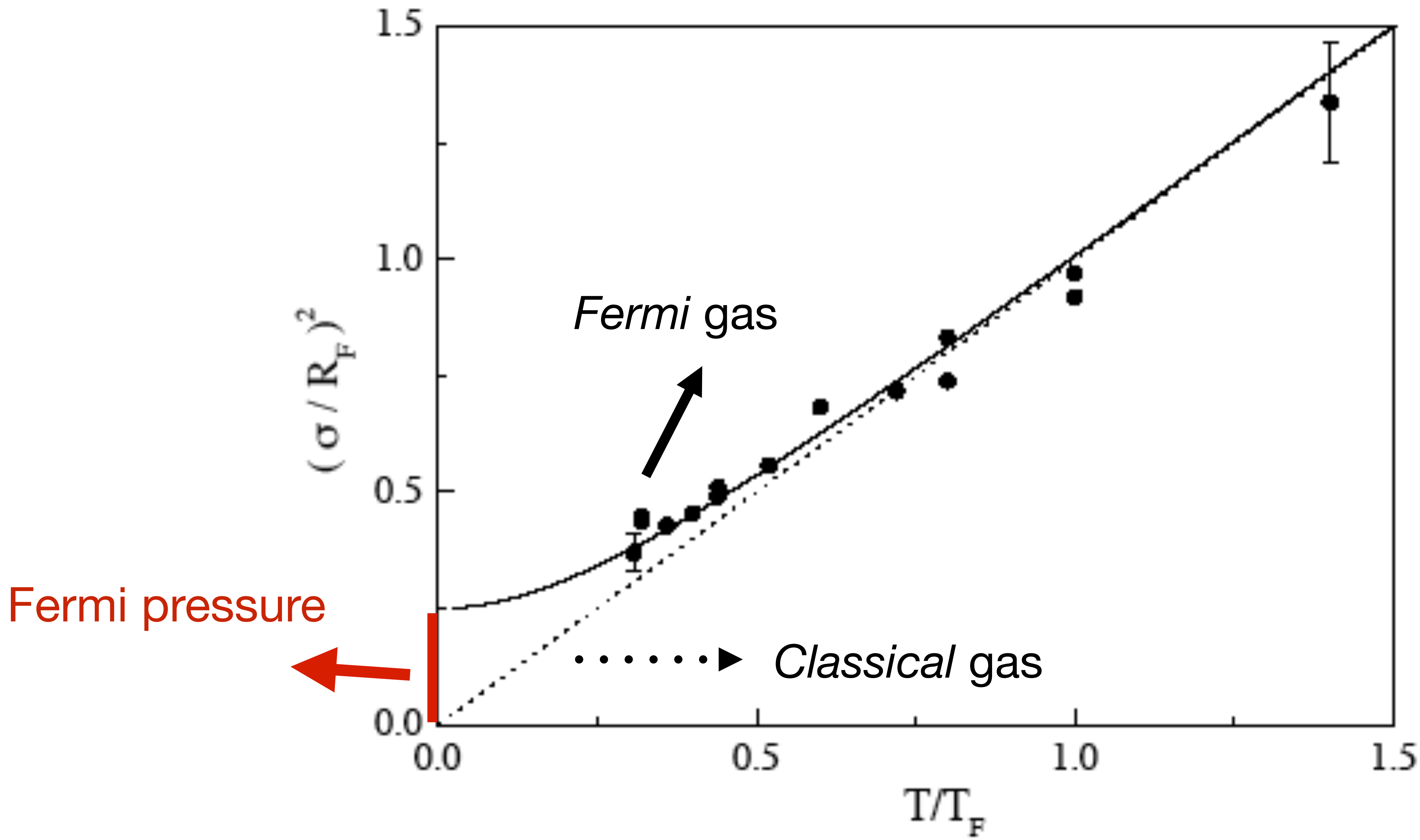
If N»1 then the Fermi radius R_F largely exceeds a_{h0}. This is another consequence of the Pauli exclusion principle.

In fact the Pauli exclusion principle induce an “effective” repulsion between the fermions in the trap -> **Fermi pressure**.

This behavior differentiates the Fermi gas respect to the Bose gas:

☑ Interacting BEC: interactions + harmonic oscillator length (*typically* $U_{int}<E_F$)

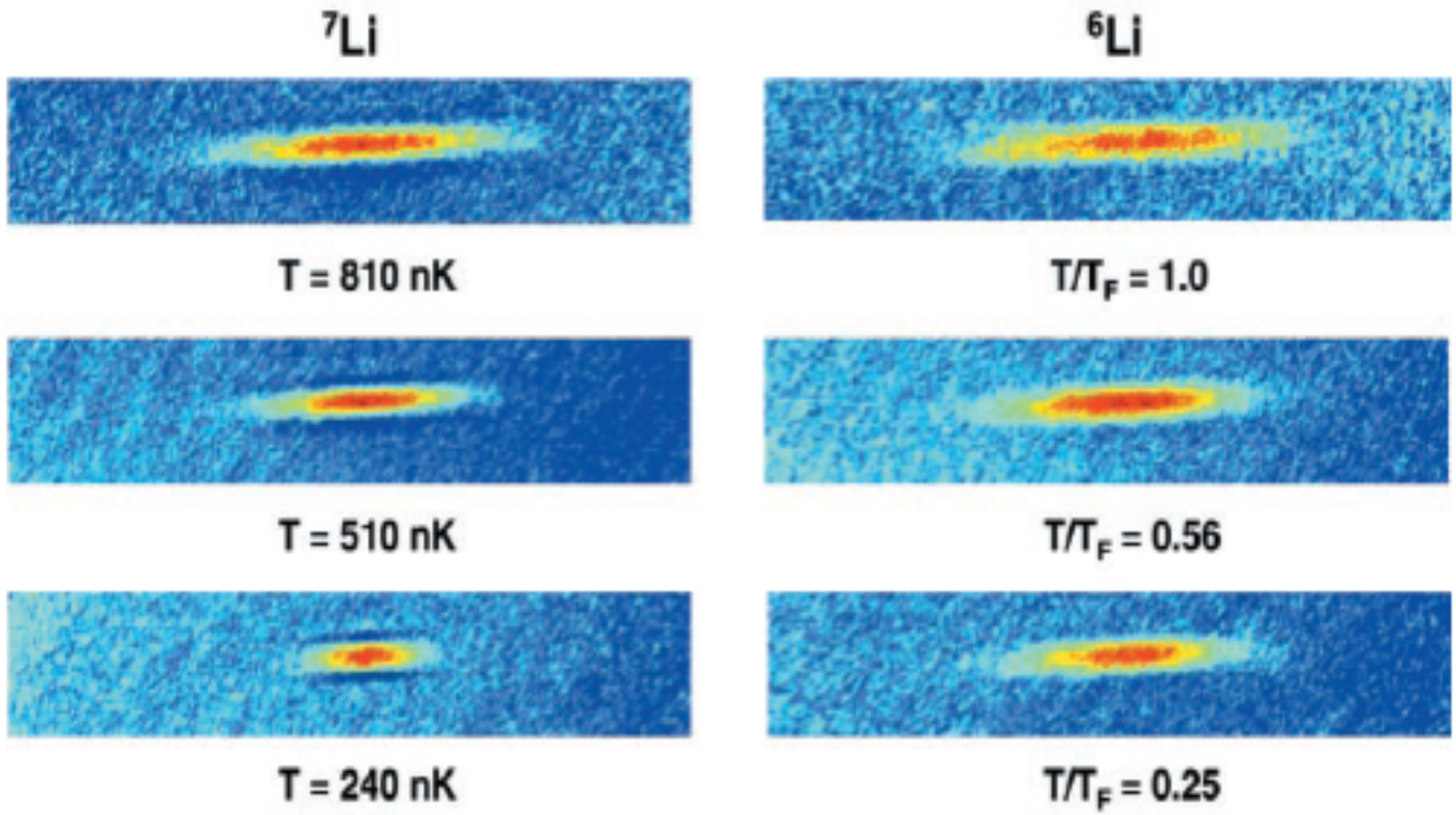
☑ Ideal BEC: harmonic oscillator length



Comparison with a classical gas: the size $R_C \rightarrow 0$ if $T \rightarrow 0$, since $R^2_C \propto T$

BEC

Fermi gas



$$R_F \propto a_{ho} \sqrt[6]{N}$$

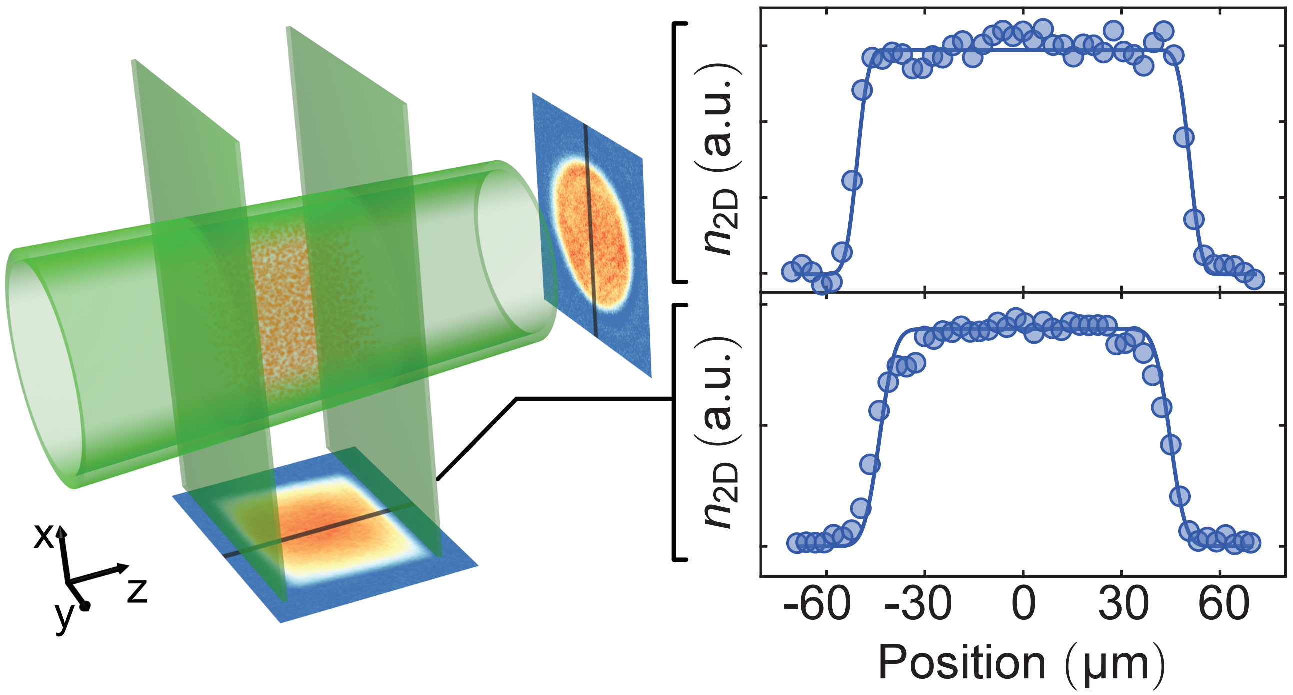
A. G. Truscott et al., Science **291**, 2569 (2001)

<u>From Wikipedia</u>

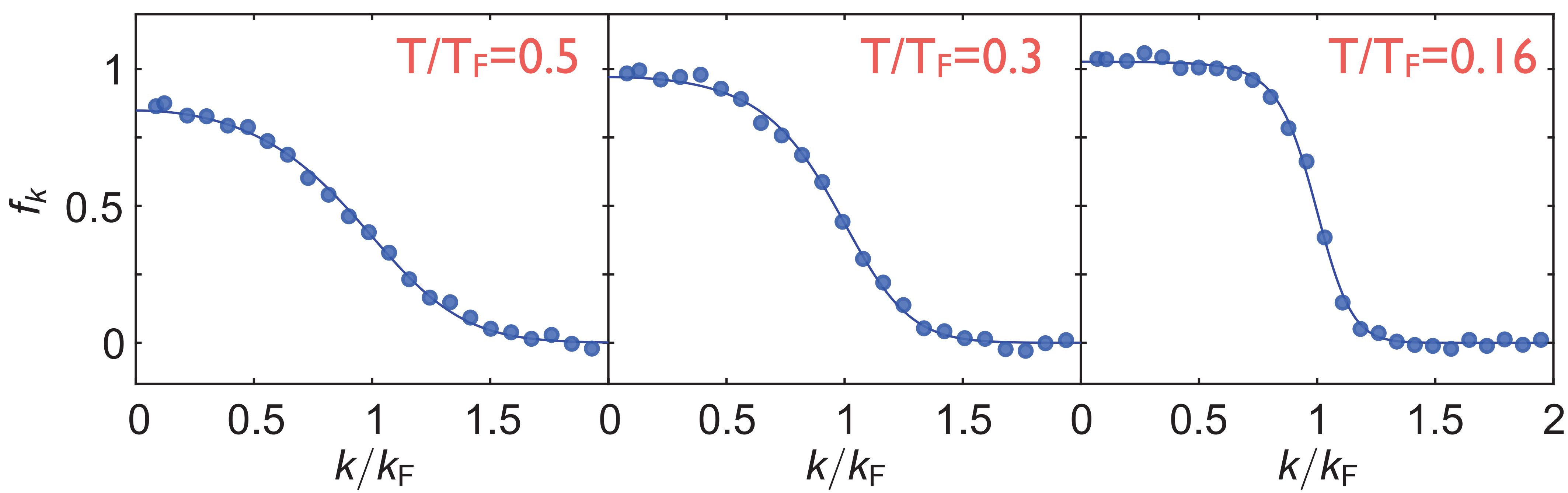
A neutron star is a type of compact star that can result from the gravitational collapse of a massive star after a supernova. Neutron stars are the densest and smallest stars known to exist in the Universe; with a radius of only about 11–11.5 km (7 miles), they can have a mass of about twice that of the Sun.

Neutron stars are composed almost entirely of neutrons, which are subatomic particles with no net electrical charge and with slightly larger mass than protons. ***Neutron stars are very hot and are supported against further collapse by quantum degeneracy pressure due to the phenomenon described by the Pauli exclusion principle, which states that no two neutrons (or any other fermionic particles) can occupy the same place and quantum state simultaneously.***

Fermions in a **flat-bottom** trap



Zwierlein's group @ MIT: realising an homogenous atomic Fermi gas



B. Mukherjee et al. Phys. Rev. Lett. **118**, 123401 (2017)