Information flow in open and composite quantum systems

Lecture IV
Composite quantum systems
Contents (Lecture IV)

• Introduction
  – Tri-partite systems

• Decoherence stabilization
  – Examples (integrable and non-integrable)
  – Depolarizing heat bath

• Dephasing coupling with RMT environment
  – This, we understand

• Dissipative Jaynes-Cummings model
  – We understand half

• General case
  – Weisskopf-Wigner theory?
Our tri-partite systems

**General model**

- **Central Quantum System**
- **Quantum Environment**
- **Heat bath \((T)\)**

**Dissipative quantum kicked rotor**
[Dittrich, Graham; Cohen; 1990 – 1997]

**Quantum chaotic environment vs.**
Collection of harmonic oscillators
[Srednicki; Cohen; Rossini et al.; Fonseca-Romero et al.; 1994 – 2008]

**RMT vs. Collection of harmonic oscillators**
[Lutz, Weidenmueller 1999]
Introduction

General model

Central Quantum System

\[ \lambda, g \]

Quantum Environment

Heat bath (T)

Dephasing

Photon exchange

\[ \sigma_x \otimes (\hat{a} + \hat{a}^\dagger) \]

RMT

Harmonic Oscillator (HO)

Kicked HO

Kicked Ising Spin chain

Quantum optical

Depolarizing

Caldeira-Leggett
Decoherence stabilization

- Jaynes-Cummings model with zero temperature heat bath
Kicked Ising spin network

Decoherence in the qubit, Measured by \textbf{Purity}

\[ P = tr \, \rho_q^2 \]
Dissipative Jaynes-Cummings

Qubit initially in sigma_x eigenstate

[Rodrigo Morfín, Master thesis (UdG, 2017)]
Decoherence stabilization

• Stabilization effect is very general
  – Results in “dynamical decoupling”

• Our aim:
  – Understand the effect
  – Make numerical predictions

• Method:
  – Simplify the heat bath, in order to obtain analytical solutions
  – Work our way towards the real heat baths

[Moreno, TG, Seligman PRA (2015);
  TG, Moreno, Seligman, Phil. Trans. R. Soc. A (2016);
  Ramírez, Moreno, TG (tbp)]
Depolarizing heat bath I

A RMT system coupled to a RMT (far) environment leads to the master equation [TG, Pineda, Kohler, Seligman, NJP (2008)]

\[ i\hbar \partial_t \rho = [H_e, \rho] - i \frac{N_e t_H \gamma^2}{\hbar} \left( \rho - \frac{1}{N} \right) \]

\[ \partial_t \rho = \frac{-i}{\hbar} [H_e, \rho] - \Gamma \left( \rho - \frac{1}{N} \right), \quad \Gamma = \frac{2\pi N_e \gamma^2}{\hbar d_f} = N_e \Gamma_{FGR} \]

\[ t_H = \frac{2\pi \hbar}{d_f} \quad \text{Heisenberg time in the far environment} \]

\[ \gamma^2 \quad \text{Modulus squared of a typical coupling matrix element} \]

\[ d_f^{-1} \quad \text{Level density in the far environment} \]

Fermi golden rule width: decay rate of a single state, Gamma: decoherence rate
Depolarizing heat bath II

A RMT system coupled to a RMT (far) environment leads to the master equation

\[ \partial_t \varrho = -\frac{i}{\hbar} [H_c, \varrho] - \Gamma \left( \varrho - \frac{1}{N_c} \right) \]

Put central system into container (add qubit)

\[ \partial_t \varrho = -\frac{i}{\hbar} [H_{qc}, \varrho] - \Gamma \left( \varrho - \text{tr}_c(\varrho) \otimes \frac{1}{N_c} \right) , \quad H_{qc} = \frac{\Delta}{2} \sigma_z + H_c + \lambda \sigma_z \otimes V_c \]

Replace uniform mixture with thermal mixture

\[ \varrho(0) = \varrho_q(0) \otimes w_T , \quad w_T = \frac{1}{Z} e^{-\beta H_c} \]

Milburn “Kicked quantized cavity mode: An open systems approach” [PRA (1987)]

\[ \frac{d \hat{\rho}(t)}{dt} = -\frac{i}{\hbar} [\hat{H}_0, \hat{\rho}] + \gamma (\phi \hat{\rho} - \hat{\rho}) \]
Dephasing coupling

- Dephasing coupling and depolarizing heat bath

\[
\frac{d}{dt} \rho_{qe} = -i [H_g, \rho_{qe}] - \Gamma \left( \rho_{qe} - \text{tr}_e(\rho_{qe}) \otimes \omega_T \right)
\]

\[
H_g = \frac{\Delta}{2} \sigma_z + H_e + g \sigma_z \otimes V_e \quad \quad \rho_q = \text{tr}_e \rho_{qe}
\]

- Only the non-diagonal element shows dynamics

\[
\rho_{21}^q(t) = e^{-i\Delta t} f_{\lambda,\Gamma}(t)
\]

\[
f_{\lambda,\Gamma}(t) = e^{-\Gamma t} \phi_{\lambda,\Gamma}(t) \quad : \quad \phi_{\lambda,\Gamma}(t) = f_{\lambda}(t) + \Gamma \int_0^t d\tau \ f_{\lambda}(t-\tau) \phi_{\lambda,\Gamma}(\tau)
\]

\[
f_{\lambda}(t) = \text{tr}_e \left[ e^{-iH_+ t} \omega_T e^{iH_- t} \right], \quad H_{\pm} = H_e \pm g V_e, \quad (\lambda = 2g)
\]

- Compare exponential (FGR) vs. Gaussian (perturbative) decay
Numerical results [PRA 2015]
Comparison with analytical formula
Limit of large dissipation

• Use Laplace transform

\[ f_{\lambda, \Gamma}(t) = e^{-\Gamma t} \phi_{\lambda, \Gamma}(t) \quad ; \quad \phi_{\lambda, \Gamma}(t) = f_{\lambda}(t) + \Gamma \int_{0}^{t} d\tau \ f_{\lambda}(t-\tau) \ \phi_{\lambda, \Gamma}(\tau) \]

\[ \Phi_{\lambda, \gamma}(s) = F_{\lambda}(s) + \Gamma \ F_{\lambda}(s) \ \Phi_{\lambda, \gamma}(s) \ , \quad F_{\lambda}(s) = \int_{0}^{\infty} dt \ e^{-st} \ f_{\lambda}(t) \]

\[ f_{\lambda, \Gamma}(t) = \frac{e^{-\Gamma t}}{2\pi i} \int_{T-i\infty}^{T+i\infty} ds \ \frac{e^{st} \ F_{\lambda}(s)}{1 - \Gamma F_{\lambda}(s)} \]

• There is one dominant pole on the real axis, near \( s \approx \Gamma \)

We expand \( F_{\lambda}(s) \) at large \( s \).

\[ F_{\lambda}(s) \sim \frac{1}{s} + \frac{1}{s^2} f'_{\lambda}(0) + \frac{1}{s^3} f''_{\lambda}(0) \]

Perturbative (Gaussian) regime:

\[ f_{\lambda, \Gamma}(t) \approx e^{-2\lambda^2 t / \Gamma} \]
Jaynes-Cummings model

• ... with the same depolarizing heat bath

\[
\frac{d}{dt} \rho_{qe} = -i \left[ H_g, \rho_{qe} \right] - \Gamma \left( \rho_{qe} - \text{tr}_e(\rho_{qe}) \otimes \omega_T \right)
\]

\[
H_g = \frac{\Delta}{2} \sigma_z + \hat{a}^{\dagger} a + g \left( \sigma_- \otimes a^{\dagger} + \sigma_+ \otimes a \right)
\]

• With the atom density matrix we find:

\[
\rho_q = \frac{1}{2} \begin{pmatrix}
1 + a_3 & a_1 - i a_2 \\
1 & 1 - a_3
\end{pmatrix}
\]

\[
a_3(t) = a_3^{(0)}(t) + \Gamma \int_0^t d\tau \left[ G(t-\tau) + F(t-\tau) a_3(\tau) \right]
\]

\[
\begin{pmatrix}
a_1(t) \\
a_2(t)
\end{pmatrix} = \begin{pmatrix}
a_1^{(0)}(t) \\
a_2^{(0)}(t)
\end{pmatrix} + \Gamma \int_0^t d\tau \begin{pmatrix}
F_2(t-\tau) & -i G_2(t-\tau) \\
i F_3(t-\tau) & G_3(\tau)
\end{pmatrix} \begin{pmatrix}
a_1(\tau) \\
a_2(\tau)
\end{pmatrix}
\]
Figura 20: $\rho_{qb}(t)$ with initial state $0.5 \sum_{k0,1} |k\rangle\langle k|$, $g = 0.02$, $\frac{\Delta-1}{2} = 0.05$
Decoherencia atómica en una zona de Ramsey fría

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...ments. If the atom-field coupling constant is much smaller than the inverse of the photon mean lifetime, the time-dependent Weisskopf-Wigner approximation is valid and allows to show the following. The atomic decoherence characteristic time is inversely proportional to the product of the square of the coupling constant and the photon mean lifetime. In other words, if the cavity quality factor is small, the atom decouples from its environment. Under these con-

\[ \frac{d\delta}{d\tau} = \frac{-2g^2}{\gamma_A} \left( 1 - e^{-\gamma_A \tau} \right) \left( \delta(\tau) - \lambda(\rho_{gg} + \rho_{eg}\rho_{ge})(\tau) \right) \]

Poster: Efraín Molano, Karen Fonseca, “QED cavities and Ramsey zones ...”
End